

# **ANGLAIS / MATHÉMATIQUES**

**SECTION EUROPÉENNE  
SESSION 2018**

CORRIGÉS

**BACCALAURÉATS GENERAL ET TECHNOLOGIQUE  
SESSION 2018**

**ÉPREUVE SPECIFIQUE MENTION « SECTION EUROPÉENNE OU DE LANGUE ORIENTALE »  
Académies de Paris-Créteil-Versailles**

**Binôme : Anglais / Mathématiques**

**CORE KNOWLEDGE – Corrigé**

**SUJET D0-32**

First part

1) Area of the trapezoid:  $\frac{1}{2}(a+b)(a+b)$

Sum of the areas of the 3 triangles :  $\frac{ab}{2} + \frac{ab}{2} + \frac{c^2}{2}$

2) Both expressions are the same :  $\frac{1}{2}(a+b)(a+b) = \frac{ab}{2} + \frac{ab}{2} + \frac{c^2}{2}$   
 $a^2+2ab+b^2 = 2ab+c^2$   
 $a^2+b^2 = c^2$

Second part

3) Hypotenuse of the triangle  $c^2 = (8\sqrt{2})^2 + (8\sqrt{2})^2$   
**c = 16 m**

Area of the square  $c^2 = 256 \text{ m}^2$

Number of guests  $\frac{256}{1.2} = 213.3$       **hence max 213 guests**

Extra question : “trapezoid” and “the Pythagorean theorem” are US English, the others are UK English.

## SUJET D0-42

### Question 1 Answer (D)

To determine the probability that a senior's name will be chosen, you must determine the total number of seniors' names that are in the lottery and divide this number by the total number of names in the lottery.

Since each senior's name is placed in the lottery 3 times, there are  $3 \times 100 = 300$  seniors' names.

Likewise, there are  $2 \times 150 = 300$  juniors' names and  $1 \times 200 = 200$  sophomores' names in the lottery.

Because the outcomes are equally likely, the probability that a senior's name will be chosen is:

$$\frac{300}{300 + 300 + 200} = \frac{300}{800} = \frac{3}{8}$$

### Question 2 Answer (C)

Statement (A) cannot be true. The perimeter of the triangle cannot be 11, since the sum of the two given sides is 11 without even considering the third side of the triangle.

Let's consider whether it is possible for the triangle to have a perimeter of 24. In this case, the third side of the triangle would be  $24 - (6 + 5) = 13$ . The third side of this triangle cannot be 13, since the sum of the other two sides is not greater than 13. By the Triangle Inequality, the sum of the lengths of any two sides of a triangle must be greater than the length of the third side. So answer (B) should be excluded.

Finally, you see that in (C), if the perimeter were 15, then the third side of the triangle would be  $15 - (6 + 5)$ , or 4. A triangle can have side lengths of 4, 5, and 6. So the perimeter of the triangle could be 15.

So the correct answer is (C).

### Question 3 Answer (C)

For 100,000 cartridges sold at \$10 per cartridge,  $s(10) = 100$  (since  $s$  is the number of cartridges sold, in thousands). Substituting into the equation yields

$$100 = \frac{3000}{2 \times 10 + a}$$

Solving this equation for  $a$  yields  $100(20 + a) = 3000$

$$20 + a = 30$$

$$a = 10$$

Now the function can be written as  $s(p) = \frac{3000}{2p+10}$ .

To determine how many cartridges will be sold at \$20 per cartridge, we need to evaluate  $s(20) = \frac{3000}{2(20)+10} = 60$ .

Since  $s$  is given in thousands, there will be 60,000 cartridges sold at \$20 per cartridge.

### Question 4 Answer (C)

Let  $a$  be the length of the side of the small square,  $b$  the length of the side of the medium square,  $c$  the length of the side of the large square.

Knowing that the area of the small square is  $6 \text{ cm}^2$ , we get  $a = \sqrt{6}$ .

Then using Pythagoras' theorem or the recap we find that  $\frac{1}{2}b = \sqrt{3}$ , so  $b = 2\sqrt{3}$ .

Likewise, we find  $\frac{1}{2}c = \sqrt{6}$ , so  $c = 2\sqrt{6}$ .

Finally the difference between the area of the medium square and the area of the large square, in  $\text{cm}^2$ , is:

$$c^2 - b^2 = (2\sqrt{6})^2 - (2\sqrt{3})^2 = 4 \times 6 - 4 \times 3 = 24 - 12 = 12.$$

## **SUJET D0-46**

1. Let's assume that there is the same probability of landing on any face whatever its shape. If this object is thrown, what do you think will be the probability of it landing on :

a) one of its square faces ?

$$\frac{6}{14}$$

b) one of its triangular faces ?

$$\frac{8}{14}$$

2. This cuboctahedron is thrown twice.

a) What is the probability of obtaining a triangular face, then a square face ?

$$\frac{12}{49}$$

b) What is the probability of obtaining two square faces ?

$$\frac{9}{49}$$

c) What is the probability of obtaining two faces of different shapes ?

$$\frac{24}{49}$$

## **SUJET D0-52**

1)  $2 \times 2.5 \times 3 + 2 \times 2.5 \times 4 = 35\text{m}^2$

2) We convert feet into metres: 3 feet = 0.9m, 7 feet = 2.10m, 6 feet = 1.80m

$$35 - 0.9 \times 2.1 - 1.8^2 = 29.87\text{m}^2$$

3)  $\frac{29.87}{2.5} = 11.948$ . She needs 12 litres of paint.

4) With the first type of can, she needs 4 cans for a price of £72.

Second type of can: 4.5litres so she needs 3 cans for a price of £60.

It's better to choose the one-gallon paint can (besides, paint will be remaining..)

5) We can find the radius of the base circle of each can:

$$4.5 \text{ litres} = 4500 \text{ cm}^3. \text{ We solve the equation } \pi r^2 \times 30 = 4500$$

We find  $r \approx 6.9\text{cm}$ . then the diameter =14cm

Each can is contained in a 14 cm<sup>2</sup> square. So she can put his 3 cans in her box (3\*14=42 cm for the length)

## **SUJET D0-53**

1) a. Mean=20.27 to 2 dp.

1) b. About 5 or 6 pupils (25-20 or 25-19)

1) c. median approx 27, LQ 21, UQ 35

1) d. UQ-LQ=14

2) a. Class range on the histogram 4cm, area of the bar 4 cm<sup>2</sup> so height 1cm

2) b. x=1, y=2, z=4, t=6, u=3

## SUJET D0-62

### Part A

1.  $5^3 = 125$  so the volume is 125 cubic feet.

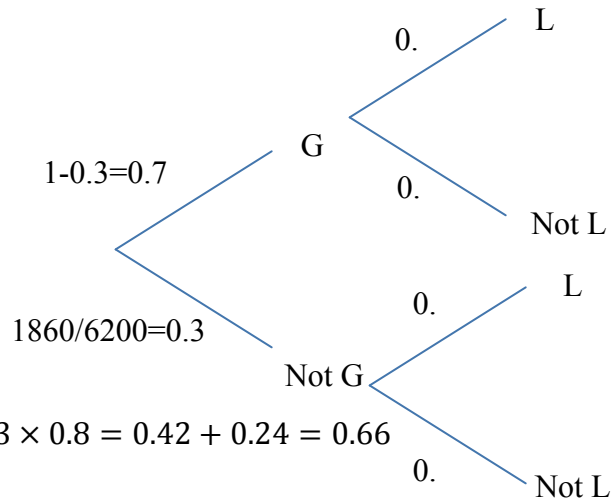
2.

a. Volume of a barge =  $100 \times 40 \times 12 = 48000$  and  $48000 \div 125 = 384$ .

b. No because 12 (the depth of a barge) is not a multiple of 5 so it's impossible to fill an entire barge with cubes whose sides are 5 feet long. The maximum number is indeed  $\frac{100}{5} \times \frac{40}{5} \times \frac{10}{5} = 20 \times 8 \times 2 = 320$ .

### Part B

1.



2.  $P(L) = 0.7 \times 0.6 + 0.3 \times 0.8 = 0.42 + 0.24 = 0.66$

3.  $0.66 \times 6200 = 4092$

4.  $4092/320=12.78$  to 2 dp so 13 barges.

## SUJET D0-65

1) Volume on one box:  $15 \times 20 \times 25 = 7\,500 \text{ cm}^3$

Volume of 28 boxes:  $28 \times 7\,500 = 210\,000 \text{ cm}^3 = 210 \text{ dm}^3 = 210 \text{ litres}$

As  $210 > 200$  he has to make two trips.

2) Converting km in miles:  $12 \text{ km} = 12 \times 1/1.6 \text{ miles} = 7.5 \text{ miles}$

19.8 miles with 1 gallon gives 7.5 miles with  $7.5 \times 1/19.8 \approx 0.4$  gallons for 1 journey there and a new 0.4 gallon for the journey back : in all, 0.8 gallons ( $0.8 \times 4.5 = 3.6$  litres)

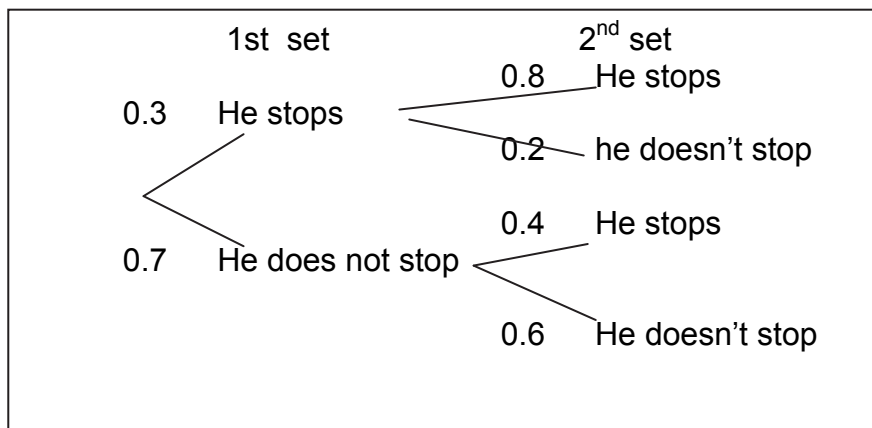
3) speed = distance / time

Distance = 12 km = 7.5 miles and time = 15 minutes = 0.25 hour

then speed =  $7.5/0.25 = 30 \text{ mph}$

4) Proba of two green lights:  $0.6 \times 0.7 = 0.42$

We multiply the probabilities written above the branches along the path



## SUJET D0-66

1. (i) Check out that  $8^2+15^2=17^2$   
(ii)  $a = 6$  and  $b = 8$ .

2. (i) No... just have to find a counter example!

(ii) Let  $(a; b; c)$  be a Pythagorean triple, and  $k$  be positive integer.  
 $a^2 + b^2 = c^2$ , hence  $k(a^2 + b^2) = kc^2$ , and  $(ka)^2 + (kb)^2 = (kc)^2$ , which means exactly that  $(ka; kb; kc)$  is a Pythagorean triplet.

(iii) There are at least as many Pythagorean triple as different values of  $k$  positive integer!

3. Let  $a, b, c, d, e, f$ , be positive integers such that:  $\left(\frac{a}{b}\right)^3 + \left(\frac{c}{d}\right)^3 = \left(\frac{e}{f}\right)^3$ .

We have then:  $\left(\frac{adf}{bdf}\right)^3 + \left(\frac{cbf}{bdf}\right)^3 = \left(\frac{ebd}{bdf}\right)^3$ , hence:  $(adf)^3 + (cbf)^3 = (ebd)^3$  .... which is impossible, since Andrew Wiles managed to prove Fermat's last theorem!

## SUJET D0-68

1) ABCDEF is a right-angled prism.

One side length of its base is 4 feet and its height is 5 feet. Its volume is 35 ft<sup>2</sup>.

- a) Volume = base area x height

$$35 = (AB \times AC / 2) \times 5$$

$$35 = AB \times 4 / 2 \times 5$$

$$35 = AB \times 10$$

$$AB = 3.5 \text{ ft}$$

- b) Pythagoras' theorem :

$$BC^2 = AB^2 + AC^2$$

$$BC^2 = 3.5^2 + 4^2$$

$$BC^2 = 28.25$$

$$BC = 5.3 \text{ (1 dp)}$$

2) Let  $a$  be the age of Jane.

Adam is  $2a$ .

Charlie is  $a - 3$ .

The sum of all their ages is  $a + 2a + a - 3 = 53$

$$4a - 3 = 53$$

$$4a = 50$$

$$a = 12.5$$

Jane is  $12 \frac{1}{2}$ .

$$3) D1 = 2+2+2 - 2 = 4$$

$$D2 = 3 + 3 + 3 - 2 = 7$$

$$D3 = 4 + 4 + 4 - 2 = 10$$

...

$$D120 = 120 + 120 + 120 - 2 = 360 - 2 = 358$$

## **SUJET D0-71**

### ANSWERS

1) A paradox is a statement that leads to a self-contradiction.

In the text, the paradox is the high level that a large part of Americans are taking and difficulties with everyday life math problems.

- The difference between "everyday math" and math at school.

- Misunderstanding/misconception in math

3) a) Bar charts. The percentage of people who have access to a phone in Arkansas (94.6%) and Connecticut (98.9%).

b) Choice B is a "good one". (The choice A is misleading, the y-axis doesn't start at 0).

4) Area of the carpet: 73 square feet = 8.03 square yards.  $8.03 \times 15 = 120,45$ .

The carpet costs £120.



## SUJET D0-73

### ANSWERS

Different methods:

$3+2=5$  and  $\frac{1kg}{5} = 200g$ . We need  $3*200 = 600g$  of apples and  $2*200 = 400g$  of sugar.

Or: let  $a$  be the quantity of apples and  $b$  the one of sugar:  $a+b=1$  kg and  $\frac{a}{3} = \frac{b}{2}$

So  $a = \frac{3}{2}b$  then  $\frac{3}{2}b + b = 1$  then  $b=2/5$  kg = 400g and  $a=1000-400 = 600g$

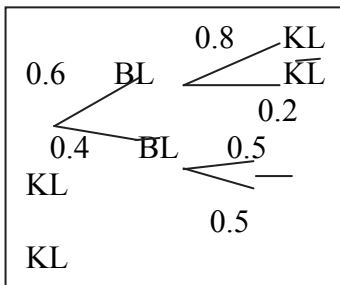
1)  $\frac{1.5}{3} = \frac{x}{2}$  thus  $x=2*1.5/3 = 1$ kg of sugar

2) The **cylinder**: Volume of the cylinder =  $\pi \times 5^2 \times 15 \approx 294 \text{ cm}^3$

Volume of the prism = area of square \* height =  $4.5 * 4.5 * 15 \approx 304 \text{ cm}^3$

3) Proba Ava likes it =  $0.6*0.8+0.4*0.5=0.68$

4)  $(15*200+12*150+12*100)/37 = 154g$  to the nearest unit.



## SUJET D0-74

1) Area of "grass zone" :

$$A_{grass} = A_{rectangle} - A_{pool} - A_{vegetables}$$

$$A_{grass} = 34 \times 15 - \pi \times \left(\frac{3.5}{2}\right)^2 - 5^2 = 475.38m^2 \text{ (to 2d.p.)}$$

1lb covers  $30m^2$ . Jonathan will need  $\frac{475.38}{30} = 16$  lbs (rounded to the upper integer).

He should buy 3 bags of 5lbs + 1 bag of 1lb.

Price of grass seeds :  $3 \times 32.5 + 1 \times 8.95 = \$106.45$

Price of the pool :  $474.99 - 10\% = 474.99 - 47.50 = \$427.49$

Total price :  $106.45 + 427.49 = \$533.94$

- 2) a)  $P=2*(34+15)=98$  meters  
b) It depends on the dimensions of the rectangle (length and width...)

### SUJET D0 81

Answer:

1)a) radius = diameter / 2 = 2.5 cm. Volume =  $\pi \times 2.5^2 \times 4 = 78.5 \text{ cm}^3$  to 1d.p.

1)b)  $5 \times 3 \times 4 = 60 \text{ cm}^3$

1)c) x = missing side of the triangle.

Pythagoras' theorem :  $2x^2=50$ , then  $x= 5$  cm. Sides of the triangle: 5; 5;  $\sqrt{50}$  cm.

Volume of the tin:  $\frac{5 \times 5}{2} \times 4 = 50 \text{ cm}^3$

2) volume of cream = y then  $\frac{5}{60} = \frac{3}{y}$  thus  $y = \frac{60 \times 3}{5} = 36 \text{ cm}^3$

3) diner so cuboids whose volume is  $60+36=96 \text{ cm}^3$  (cake + cream).

Then 2 eggs per cupcake consequently  $2 \times 20 = 40$  eggs per 20 cupcakes.

### SUJET D0 82

**Solution**

1) a) 1 foot = 30.48 centimetres and 1 foot = 12 inches then 12 inches = 30.48 cm then **1 inch = 2.54 cm.**

b) 9 feet and 7 ½ inches =  $9 \times 30.48 + 7.5 \times 2.54 = \mathbf{293.4 \text{ cm}}$  to 1dp  
7 feet and 9 ¼ inches =  $7 \times 30.48 + 9.25 \times 2.54 = \mathbf{236.9 \text{ cm}}$  to 1dp

2) a) Let d be the distance between the bull and the floor. The wall is perpendicular to the floor, the Pythagorean theorem gives :

$$d^2 = 293.4^2 - 236.9^2$$

$$\mathbf{d = 173 \text{ cm}}$$

b)  $172.72 / 30.48 = 5.7$  (to 1dp)

$$172.72 \text{ cm} = 5 \times 30.48 \text{ cm} + 20.32 \text{ cm}$$

$$172.72 \text{ cm} = 5 \text{ feet} + 20.32 \text{ cm}$$

But  $20.32 / 2.54 = 8$  then 20.32cm is equal to 8 inches

$$\mathbf{172.72 \text{ cm} = 5 \text{ ft. } 8 \text{ in.}}$$

3) the player randomly throws the dart, then :

$$P(\text{"the dart touches the bull"}) = \frac{\text{area of the bull}}{\text{area of the dartboard}}$$

$$P(\text{bull}) = \frac{\pi \times \left(\frac{1.4}{2}\right)^2}{\pi \times \left(\frac{34}{2}\right)^2}$$

$$\mathbf{P(\text{bull}) = 0.0017} \text{ to 4dp.}$$

## SUJET D0 83

1. 2017 IAAF world championships marathon in London :
  - the total length of the course is 26.2 miles, and since 1 mile represents 1,609.34 meters, the total length of the course is also  $26.2 \times 1,609.34 = 42,164.708$  meters or 42.165 kilometers, rounded to the nearest meter.
  - the length of one lap is 10 kilometers, or 10,000 meters, that is to say  $10,000 \div 1,609.34 = 6.214$  miles, rounded to 3 decimal places.
  - The course includes four laps of 10 kilometers which represent a total of 40 km. The remaining  $42.165 - 40 = 2.165$  km are run between the start and finish point and the lap turn point. That route is used twice during the course (once at the beginning and once at the end) so it's length is  $2.164708 \div 2 = 1.082$  kilometers, rounded to the nearest meter.
2. During the 26.2 mile men's marathon (corrected to the nearest tenth) at the 2012 Summer Olympics:
  - 85 runners have run the full 26.2 miles, which makes a total amount of  $85 \times 26.2 = 2,227$  miles. 20 runners did not finish, so each have run between 0 and 26.2 miles, which makes a total amount between 0 and  $20 \times 26.2 = 524$  miles for those 20 runners. So all the participants together have run between 2,227 miles and  $2,227 + 524 = 2,751$  miles, which makes 2,800 miles corrected to 2 significant figures, and in standard form also corrected to 2 significant figures  $2.8 \times 10^3$  miles.
  - Among the 85 participants, the winner needed 2 hours 8 minutes and 1 second that is to say  $2 \times 3,600 + 8 \times 60 + 1 = 7,681$  seconds, and the slowest one 2 hours 55 minutes and 54 seconds, that is to say  $2 \times 3,600 + 55 \times 60 + 54 = 10,554$  seconds. The other 83 needed a duration in between. The lower bound of the total duration of the 85 courses is  $84 \times 7,681 + 10,554 = 655,758$  seconds. Since there are  $3,600 \times 24 = 86,400$  seconds in a day, that lower bound is also  $655,758 \div 86,400 = 7.6$  days rounded to 1 decimal place. The upper bound of the total duration of the 85 courses is  $7,681 + 84 \times 10,554 = 894,217$  seconds, or  $894,217 \div 86,400 = 10.3$  days rounded to 1 decimal place. So if all the 85 participants who finished the course had done it one at a time without any break, it would have taken between 7.6 and 10.3 days.

## SUJET D0 84

### ANSWERS

1.
  - a) Use your calculator to work out  $\pi - 3.14$  and  $22/7 - \pi$  correct to 5 dp.  
 **$\pi - 3.14 = 0.00159$  to 5dp       $22/7 - \pi = 0.00126$  to 5 dp**
  - b) Daniel Tammet chose to recite  $\pi$  on Pi Day (March 14th, 3rd month 14th day). Why do you think some mathematicians believe Pi Day should be July 22nd ?  
**As  $\pi - 3.14 > 22/7 - \pi$  then 22/7 is a better approximation of  $\pi$ .**
2.
  - a) To this day more than 22 trillion digits of  $\pi$  have been discovered. An average person can read out approximately 120 digits/min. Keeping this pace, how long would it take to recite these digits ?

**348,808 years (correct to 6 sf)**

- b) Assuming a total world population of roughly 7 billion people, how many digits of  $\pi$  everyone would have to memorize in order to preserve all known digits of  $\pi$  in our collective heads ?

**1,428 digits**

3. Let's view  $\pi$  as a big, random string of numbers, then we know the odds of finding any string of digits in the first 100 million digits of  $\pi$ :

Number length	1-5	6	7	8	9	10	11
Chance of finding	100 %	nearly 100 %	99.995 %	63 %	9.5 %	0.995 %	0.09995 %

- a) If we search for the digit "6" in  $\pi$ , what is the chance that a digit picked at random in the first 100 million decimals of  $\pi$  is equal to "6" ?

**1/10**

- b) If we search for the string of digits "61" in  $\pi$ , what is the chance that a string of two digits picked at random in the first 100 million decimals of  $\pi$  is equal to "61" ?

**1/100**

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**Domaine 1 – MAPPING – corrigés**

<b>Sujet : D1_41</b>
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(a) Two points for C : (1,9) and (5,0)

$$C(1) = 9, \text{ gives } b = 9 ; C(5) = 0 \text{ so : } 16a + 9 = 0 ; a = -\frac{9}{16} = -0.5625$$

Eventually we find that  $C(x) = -0.5625(x-1)^2 + 9$ .

(b)  $C(0) = 135/16 = 8.4375$ .

$$H(0) = 8$$

So CHELSEA

(c) Value of x for which  $H(x) = 0$ .

$$-\frac{2}{9}x^2 + 8 = 0 .$$

$x=6$  since x is positive .

(d) domain of C : x between 0 and 5 ; range of C : C(x) between 0 and 9

domain of H : x between 0 and 6 ; range of H : H(x) between 0 and 8

(e) They have the same height at an horizontal distance of 3.7m

Questions supplémentaires :

- Common height ?  $H(3.7) \approx 4.96\text{m}$ .

- Could you solve  $C(x) = H(x)$  ? Equation :  $\frac{49}{144}x^2 - \frac{9}{8}x - \frac{7}{16} = 0$

$\Delta \approx 1.86 ; x \approx 3.7$

**Sujet : D1\_43**

a) x coordinate of the vertex : 2

So axis of symmetry  $x=2$  . As  $(0,2)$  belongs to the curve, then  $(4,2)$  also.

b)  $h(t) = at^2+bt+c$

$h(0)=2$  gives  $c=2$

$h(4)=2$  gives  $16a+4b+2=2$  hence  $4a+b=0$  ( $L_1$ )

$h(3)=20$  gives  $9a+3b+2=20$  hence  $3a+b=6$  ( $L_2$ )

$(L_1)-(L_2)$  gives  $a=-6$  hence  $b=24$ .

c) x coordinate of the vertex is 2 hence max :  $h(2) = 26$ . So NO.

d) We have to solve  $h(t)=0$

$\Delta = 624$  Only positive root :  $\frac{-24+\sqrt{624}}{-12} \approx 4.08$

**Sujet : D1\_52**

**Answer:**

1)  $(0,1)$ ,  $(1,6)$ , and  $(2,7)$

2)  $h(0)=1$  gives  $c=1$

$h(1)=6$  gives  $a + b + 1 = 6$

and  $h(2)=7$  gives  $4a + 2a + 1 = 7$

Solving the two simultaneous equations, we get  $a = -2$  and  $b = 7$

3) The trajectory is a parabola hill-shaped (since  $a$  is negative), its vertex is  $(1.75, 13.25)$  or  $(7/4, 53/4)$ .

It has an axis of symmetry :  $x=7/4$  and its maximum height is 13.25m

4)  $h(3)=4$

then yes given that the part of the window which is open is between 3m80 and 4m30.

1)



Radius	7 inches	6 inches	3 inches
Crust Thickness	1-inch	1-inch	1-inch
Area, Total	$\pi \times 7^2 = 49\pi \text{ in.}^2$	$\pi \times 6^2 = 36\pi \text{ in.}^2$	$\pi \times 3^2 = 9\pi \text{ in.}^2$
Area, Inside	$\pi \times (7-1)^2 = 36\pi \text{ in.}^2$	$\pi \times (6-1)^2 = 25\pi \text{ in.}^2$	$\pi \times (3-1)^2 = 4\pi \text{ in.}^2$
Area, Crust	$\pi \times 7^2 - \pi \times 6^2 = 13\pi \text{ in.}^2$	$\pi \times 6^2 - \pi \times 5^2 = 11\pi \text{ in.}^2$	$\pi \times 3^2 - \pi \times 2^2 = 5\pi \text{ in.}^2$

2)

<p><b>a.</b> Write functions for the total area, the inside area, and the crust area in terms of the radius, <math>r</math>.</p> $T(r) = \pi r^2,$ $I(r) = \pi(r-1)^2$ $= \pi(r^2 - 2r + 1)$ $= \pi r^2 - 2\pi r + \pi,$ $C(r) = T(r) - I(r)$ $= \pi r^2 - (\pi r^2 - 2\pi r + \pi)$ $= 2\pi r - \pi.$	<p><b>b.</b> For what size pizza is the inside area equal to the crust area...and would you order it?</p> $I(r) = C(r),$ $\pi r^2 - 2\pi r + \pi = 2\pi r - \pi,$ $\pi r^2 - 4\pi r + 2\pi = 0,$ $r^2 - 4r + 2 = 0.$ <p>By the quadratic formula, <math>r = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 2}}{2 \times 1} = 2 \pm \sqrt{2}.</math>          Since <math>r \geq 1</math>, this means <math>r = 2 + \sqrt{2} \approx 3.41</math> inches. But who would order a pizza that's half crust??</p>
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**3)a)** crust : pizza =  $13\pi : 49\pi = 13 : 49 \approx .265$

**3)b)** we want to find  $r$  such that  $\pi(2r - 1) : \pi r^2 = 1 : 3$

$$\frac{\pi(2r - 1)}{1} = \frac{\pi r^2}{3}$$

$$r^2 - 6r + 3 = 0$$

2 roots 0.6 and 5.5

But  $0.6 < 1$ : impossible since the crust is 1 inch wide, so  $r = 5.5$  inches.

**Sujet : D1\_ 62**

**Domain 1**

1- From the text :  $C(0) = 2750$  so  $c = 2750$

$$C(10) = 4250 \text{ and } C(20) = 8550$$

Simultaneous equations solved by elimination:

$$\begin{cases} C(10) = 4250 \\ C(20) = 8550 \end{cases} \text{ iff } \begin{cases} 100a + 10b + 2750 = 4250 \\ 400a + 20b + 2750 = 8550 \end{cases} \text{ iff } \begin{cases} 10a + b = 150 \\ 20a + b = 290 \end{cases} \text{ iff } \begin{cases} 10a + b = 150 \\ 10a = 140 \end{cases} \text{ iff } \begin{cases} b = 10 \\ a = 14 \end{cases}$$

2- Profit = Revenue – Cost

3- Solve  $P(x) \geq 0$ . As  $\Delta = 560^2 - 4 \times (-14) \times (-2750) = 159600$

So there are two roots that are 5.73 and 34.26 to 2 decimal places.

So the profit is positive when the number of containers shipped is between 6 and 34.

4- Abscissa of the vertex : 20 maximum profit:  $P(20)=2850$  pounds

5- Vocabulary about how to graph a function...parabola, hill shaped...

**Sujet : D1\_ 63**

1) Obvious! (for the product we can use the difference of two squares

2)

a)  $(-4; 7)(4; -7)(-2; 14)(2; -14)(1; -28)(-1; 28)$

b)  $(4, -7)$

c)  $(4, -7)$

d)  $(8, 9)$

e)  $(-4, -9)$

3) a)  $x^2 - 2x - 15 = 0$

b)  $\Delta = 64$  two solutions:  $-3$  and  $5$

c)  $-3$  and  $5$

**Sujet : D1\_ 64**

1)  $x = 100$ . Cost =  $C(100) = 3\,200$  pounds.

The receipt is  $18 \times 100 = 1\,800$  pounds then the factory doesn't make any profit.

It's a loss of  $3200 - 1800 = 1\,400$  pounds

2)  $C$  is a quadratic function.

The curve is a parabola hill-shaped since the coefficient of  $x^2$  is negative.

The vertex has for  $x$ -coordinate  $-b/2a = -30/(2 \times -0.1) = 150$ . And its  $y$ -coordinate is

$C(150) = 3450$ . Axis of symmetry :  $x = 150$ . The highest expense is for 150 umbrellas produced and the expense is £3 450.

3)  $R(x) = 18x$ .

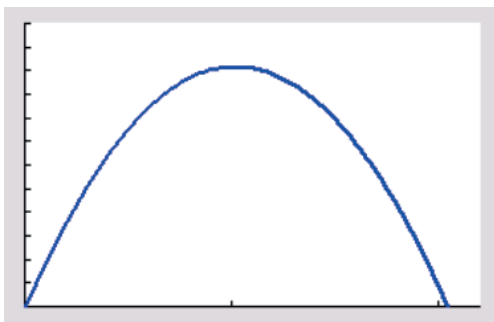
$R$  is a linear function. Its curve is a straight line which passes through  $(0,0)$  and  $(100, 1\,800)$



- 4)  $P(x) = R(x) - C(x) = 0.10x^2 - 12x - 1200$
- 5) The factory earns money when the profit is positive.  
 Discriminant  $\Delta = (-12)^2 - 4 \cdot 0.1 \cdot (-1200) = 624$   
 Roots:  $\approx -64.9$  and  $\approx 184.9$ .  
 Sign of this quadratic: negative between the roots  
 Then the factory makes a profit from 185 umbrellas produced.
- 6) X-coordinate of the vertex:  $12/0.2 = 60$ .  
 The parabola giving the profit increases from 60 to 300, so highest profit for 300 umbrellas produced and value of the profit:  $P(300) = 4200$  pounds.

**Sujet : D1\_65**

- 1) Solve equation  $h = 0 \Leftrightarrow t(20 - 9.8t) = 0 \Leftrightarrow t = 0$  or  $t = \frac{20}{9.8} \approx 2.041$  seconds.
- 2) First, let's solve  $h = 5$ ; we find  $b^2 - 4ac = 204 > 0$  therefore we get two solutions :  
 $t_1 \approx 0.292$  and  $t_2 \approx 1.749$ . The stone is more than 5 meters above the ground for  
 $t_2 - t_1 \approx 1.457$  seconds.
- 3) Solve  $h > 12$ :  $\Delta = -70.4 < 0$ , there is no solution. Answer is **NO**.
- 4) coordinates of the vertex:
- $x_V = -\frac{b}{2a} = -\frac{20}{2(-9.8)} = \frac{10}{9.8} \approx 1.02$
  - $y_V = f(x_V) \approx 10.20$
  - the y-coordinate of the vertex corresponds to the maximum value of the quadratic function  $h$  because the leading coefficient  $a = -9.8 < 0$  (the parabola opens down).
- 5) When sketching the graph, the student is expected to draw:
- the y-intercept (0);
  - the x-intercepts (0 and 2.041), solutions to  $h = 0$ ;
  - the coordinates of the vertex (1.2; 10.20);
  - the solutions to  $h = 5$  (0.292 and 1.749);
- She should also explain why the parabola opens down (the leading coefficient  $a = -9.8 < 0$ ).

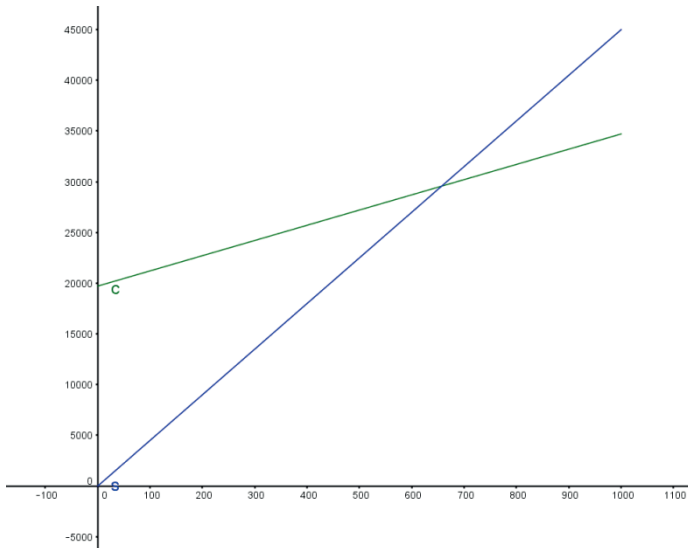


Xmin=0  
 Xmax=2.2  
 Xgrad=1  
 Ymin=0  
 Ymax=12  
 Ygrad=1

1.  $C(x) = 19170 + 15x$ .

2.  $S(x) = 45x$

3. Graph



4. The break-even point occurs when expenses (total cost of manufacturing calculators) equal the selling price (money received from sale of calculators). Therefore, the company is neither making a profit nor running at a loss.

(639,28755)

5. This portion of the graph represents expenses (total cost of manufacturing the calculators) being greater than the selling price (money received from sale of calculators). Therefore, the company is making a loss.

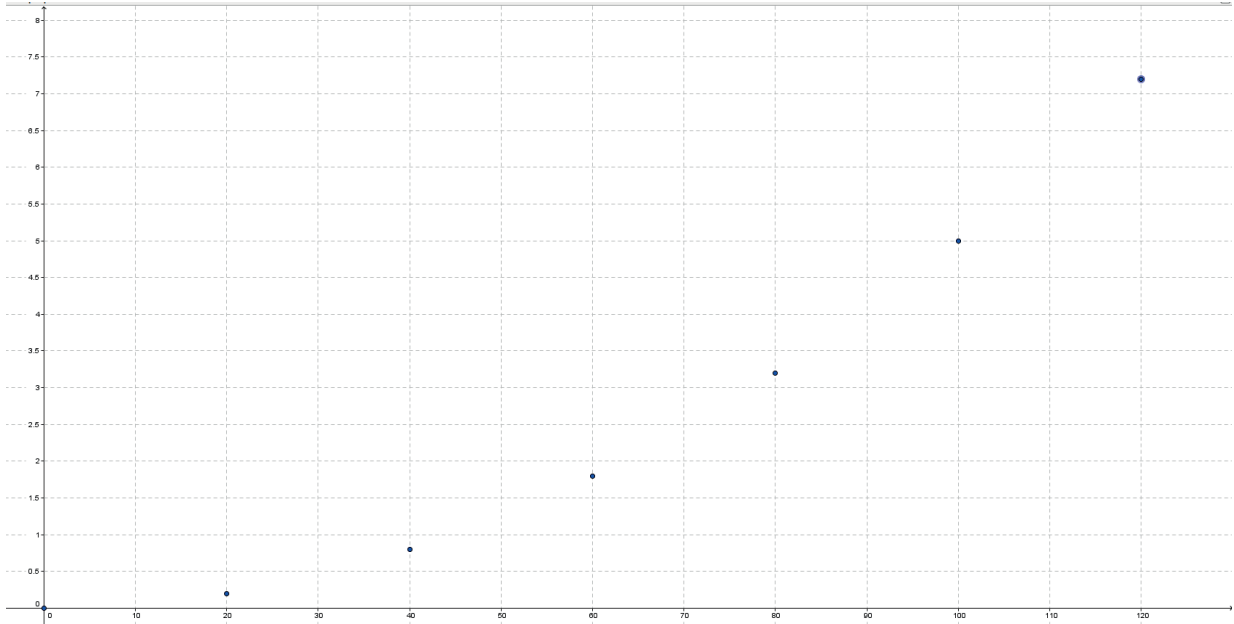
6.  $P(x) = 30x - 19170$ .

7. Determine whether a profit or loss is made when:

a. -7170 (a loss of £7170)

b. 4830 (a profit of £4830)

1.



2.  $d$  is not a linear function :  $\frac{0.2 - 0}{20 - 0} \neq \frac{0.4 - 0}{40 - 0}$ .

We can conjecture that  $d$  is a quadratic function

$$3. \begin{cases} d(0) = 0 \\ d(20) = 0.2 \\ d(40) = 0.8 \end{cases} \Leftrightarrow \begin{cases} c = 0 \\ a \times 20^2 + b \times 20 + c = 0.2 \\ a \times 40^2 + b \times 40 + c = 0.8 \end{cases} \Leftrightarrow \begin{cases} c = 0 \\ 400a + 20b = 0.2 \\ 1600a + 40b = 0.8 \end{cases} \Leftrightarrow \begin{cases} c = 0 \\ b = 0 \\ a = \frac{1}{2000} \end{cases}$$

$$d(t) = \frac{t^2}{2000}$$

4. It does

$$5. \text{ Solve } d(t) = 100 \Leftrightarrow \frac{t^2}{2000} = 100 \Leftrightarrow t^2 = 2000 \times 100 \Leftrightarrow t^2 = 200,000 \Leftrightarrow t = \sqrt{200,000} = 447$$

It reaches the floor after 0.447 s

A company manufactures and sells  $x$  cheap radios per month.

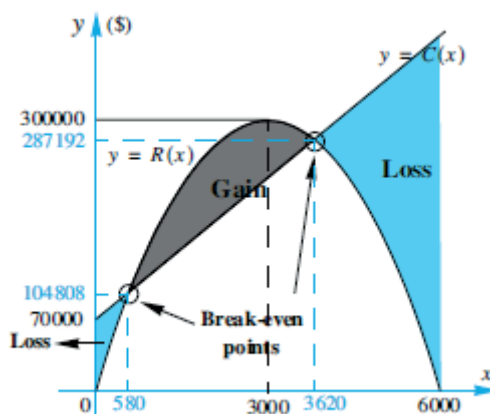
The cost,  $\$C$ , involved in producing  $x$  radios per month is given by the equation

$$C = 60x + 70000, \quad 0 \leq x \leq 6000$$

The revenue equation,  $\$R$ , based on the sales of  $x$  radios per month is given by the equation

$$R = -\frac{1}{30}x^2 + 200x, \quad 0 \leq x \leq 6000$$

1. Accurately draw the graphs of the cost and revenue functions on the same set of axes.



2. Calculate
  - a. the minimum cost involved ?  **$\$70,000$**
  - b. the maximum revenue ? vertex:  $x = -b/(2a) = 3000$  and  $y = \$300,000$ : **max revenue**

3. Why is there a cost involved when no radios are produced ?

**Fixed cost**

4. On your graph, identify the break-even points.

**See graph in (a)**

5. What profit does the company when 2000 radios are produced and sold ?

**$\$76667$**

6. a) Find an expression in terms of  $x$  for the profit,  $\$P$ , this company makes on the sales of their radios.

$$P(x) = 140x - \frac{1}{30}x^2 - 70000, \quad 0 \leq x \leq 6000$$

- b) How many radios would they need to sell to earn 60000\$ ?

$\Delta = 2266$  so two roots **1385 and 2814 radios**

- c) How many radios would they need to sell to achieve this maximum profit.

$-b/(2a) = 2100$

- d) What is the maximum profit the company can hope to make ?

77000 dollars

7. For what values of  $x$  will the company be in the red ?

$$P(x) < 0 \text{ for } x \leq 580 \text{ or } x \geq 3620$$

**Sujet : D1\_ 73**

**Solution :**  $R = \frac{x}{2}$

1)  $Perimeter = y + x + y + \frac{1}{2} \times 2\pi \times \frac{x}{2} = x + 2y + \pi x = 2y + x + 1.57x = 2y + 2.57x$

now  $Perimeter = 20$

then  $2y + 2.57x = 20$

thus  $y = 10 - \frac{2.57}{2}x$

i.e.  $y = 10 - 1.285x$ .

2)  $A(x) = A_{rectangle} + A_{semicircle} = x \times y + \frac{1}{2}\pi \left(\frac{x}{2}\right)^2$

$A(x) = x(10 - 1.285x) + 1.57 \times \frac{x^2}{4}$

$A(x) = 10x - 1.285x^2 + 0.3925x^2$

$A(x) = 10x - 0.8925x^2$

3)  $A(x)$  is a quadratic function with  $a = -0.8925$  ;  $b = 10$  ;  $c = 0$ .

$a < 0$  so its graph is a parabola which opens down. It has a maximum which occurs at

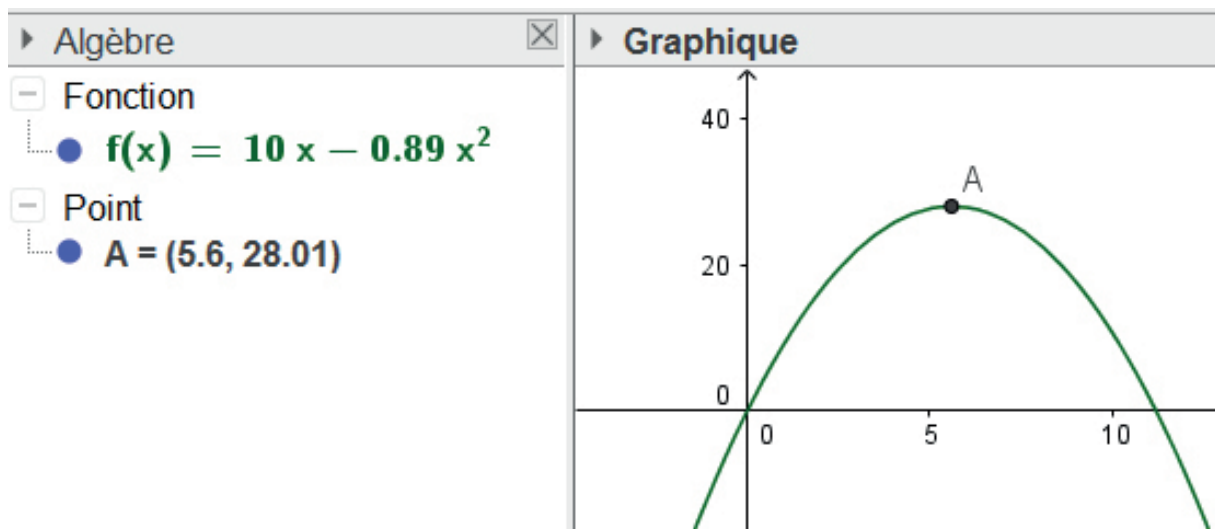
$x = \frac{-b}{2a} = \frac{-10}{2 \times (-0.8925)} \approx 5.6$  (to 1d.p.)

The area is maximum when  $x = 5.6$  ft.

4)  $x = 5.6$  and  $y = 10 - 1.285x$  then  $y = 2.8$  ft.

$A(5.6) = 10 \times 5.6 - 0.8925 \times 5.6^2 \approx 28$

The maximum area is 28 ft<sup>2</sup>.



**Sujet : D1\_ 74**

1. (i)  $f$  is an linear function: its graph is a straight line, with  $y$ -intercept equal to  $-1$ , and a positive gradient equal to  $2$ .  
 (i)  $f$  is an increasing function, whose range is  $[f(0); f(6)]$ , i.e.  $[-1; 11]$ .
  
2. (i) You can use the discriminant or complete the square.  
 $g(x) = -1$  for  $x = 2$ , and  $g(x) = 3$  for  $x=0$  or  $x=4$ .  
 (ii)  $f$  is a quadratic function: its graph is a parabola turning upward ( $a$  is positive),  
 with vertex  $S(2; -1)$  and  $y$ -intercept  $+3$ .  
 (iii) Sketch roughly the graph of the function: its range is  $[-1; +\infty[$ .
  
3. You can use your graph to have an idea of what's going on, then solve the equation:  

$$x^2 - 4x + 3 = 2x - 1.$$
 The solutions are  $3 \pm \sqrt{5}$ , which are in both domains.
  
4. (i) The graph of function  $h$  is a parabola passing through point  $A(1; 7)$  and with vertex  $S(k; 5)$ ; the sign of  $a$  should be positive... There are infinitely many such functions  
 (use a graph!).  
 (ii) We know that  $h(x) = a(x - \alpha)^2 + 5$ , with  $a$  positive and  $h(1) = 7$ .  
 Hence,  $a(1 - \alpha)^2 = 2$ :  $a = 2$  and  $\alpha = 0$  for instance, etc....

**Sujet : D1\_ 81**

- 1) The following function maps an element  $x$  onto its image  $f(x) = y$ .  
 $f : x \rightarrow 2x - 3$ .  
 (a) Since the gradient  $a$  is positive,  $f$  is an increasing function, so the range is  $\{f(0) \leq y \leq f(4)\} = \{-3 \leq y \leq 5\}$  or with the interval notation  $[-3,5]$ .  
 (b)  $f$  is a one-to-one function because each image has one pre-image as we can see when we solve  $2x - 3 = y$  for  $x$ :  $2x = y + 3$  therefore  $x = \frac{y+3}{2}$
  
- 2) (a) The graph is a parabola, it is U-shaped. The equation of the axis of symmetry is  $x=0$ .
  
- 2) (b) (i) The range has no negative number because the square of a real number is greater than or equal to  $0$ .  
 (ii) Any number which is not  $0$  has two pre-images.  
 $4$  and  $-4$  both map onto  $16$ .  
 (iii) Of course the function is many-to-one : actually it is two-to-one (except for  $0$ ).
  
- 3) (a)
 

$x$	$0$	$2$	$4$
$y$	$-3$	$1$	$5$

The equation is  $y = 2x - 3$

At least 2 points. This is a straight line which passes through these points.

(b)  $f(12.5) = 25 - 3 = 22 \neq 21$  so the point doesn't lie on the graph.

**Sujet : D1\_ 82**

1)a) straight line passing through A(1,4) and B(3,2). Gradient  $m = \frac{4-2}{1-3} = -1$

$S(t) = -t + p$  with  $S(1) = -1 + p = 4$  then y-intercept =  $p = 5$ :  $S(t) = -t + 5$ .

1)b)  $S(0)=5$  m

1)c)  $S(t)=0$  when  $t=5$ s

2)a) quadratic function so  $B(t)=at^2+bt+c$  with  $B(0)=1$ ;  $B(2)=5$  and  $B(3)=4$

You can check that the given function satisfies the three conditions or you can solve simultaneous equations:

$$\begin{cases} c = 1 \\ 4a + 2b + c = 5 \\ 9a + 3b + c = 4 \end{cases} \begin{cases} c = 1 \\ 4a + 2b = 4 \\ 9a + 3b = 3 \end{cases} \begin{cases} c = 1 \\ 2a + b = 2 \quad E2 \\ 3a + b = 1 \quad E3 \end{cases} \quad \begin{array}{l} E3-E2 \text{ gives } a=-1 \text{ then } b=4 \end{array}$$

Then  $B(t) = -t^2 + 4t + 1$

2)b) Hill shaped parabola since  $a < 0$ . X-coordinate of the vertex:  $-b/(2a)=2$

At 2 s the ball is the highest

2)c)  $B(t)=0$ . Discriminant = 20, then 2 solutions:  $\frac{-4 \pm \sqrt{20}}{-2} = 2 \pm \sqrt{5}$ . Only one solution is positive:  $2 + \sqrt{5} \approx 4.2$  to 1d.p.

The ball reaches the ground at roughly 4.2s after the beginning of the throwing.

3)Equation  $S(t)=B(t)$

$t^2 - 5t + 4 = 0$ . Discriminant = 9 ; Two solutions : 1 and 4s. Then they have the same height at 1 and 4 s after the throwing.

**Sujet : D1\_ 83**

1)  $30 \times 16 = 480$  . The area of the garden is 480 square yards.

2)  $2 \times (30 \times x + (16 - 2x) \times x) = 2 \times (30x + 16x - 2x^2) = -4x^2 + 92x$

3) Half of the garden area is 240.

We have to solve  $-4x^2 + 92x = 240$

$$4x^2 - 92x + 240 = 0$$

$$x^2 - 23x + 60 = 0$$

4)  $\Delta = 289$  so,  $\text{sqrt}(\Delta) = 17$

roots of the quadratic equation:

$$x_1 = 20 \quad \text{and} \quad x_2 = 3$$

$x_1$  is not valid for this problem. So the solution is  $x = 3$ .

The width of the path must be 3 yards.

5) Area of the path:  $30x + 16x - x^2 = 46x - x^2$

Equation  $46x - x^2 = 240$

Discriminant : 1156 two solutions 6 and 40. By 40 is impossible ( $40 > 16$ ).

The width of the path must be the double: 6 yards.

<b>Sujet : D1_84</b>
----------------------

1. (a) Parabola, U-shaped so minimum.

The minimum of function  $C$  occurs for  $q_1 = \frac{-b}{2a} = 500$  units.

(b)  $B(q) = 5q - C(q) = 5q - (0.01q^2 - 10q + 2510) = -0.01q^2 + 15q - 2510$ .

(c) Parabola, hill-shaped so maximum. The maximum of function  $B$  occurs for

$$q_2 = \frac{-b}{2a} = 750 \text{ units:}$$

hence the benefit is not maximum when the cost of production is minimum.

2. *Company A: For 500g, price is  $5 * 0.9 = 0.45$  pounds*  
*Company B: For 5 pounds, weight is  $500 * 1.1 = 550$  grams*

The price per kilo for company  $A$  is 9 pounds, while the price per kilo for company  $B$  is 9.0909...

3.  $f(x)$  can be written:  $f(x) = kx^2 + 2$ , and we know that  $f(3) = 3^2 = 9$ .

Hence  $9k + 2 = 9$  and  $k = \frac{7}{9}$  which leads to the final expression of  $f(x)$ :

$$f(x) = \frac{7}{9}x^2 + 2.$$



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**Binôme : Anglais / Mathématiques**

Corrigés

## D2 Differentiation

### Corrigé de D2 – 71

1. We use Pythagoras theorem and get  $\sqrt{4 - x^2}$
2.
  - a. The area of the triangle is  $A(x) = \frac{x\sqrt{4-x^2}}{2}$ .
  - b.  $x$  is a length, therefore it must be positive. Moreover  $4 - x^2 \geq 0 \Leftrightarrow -2 \leq x \leq 2$ . Thus  $A(x)$  is defined on  $[0;2]$
  - c.  $\frac{dA}{dx} = \frac{2-x^2}{\sqrt{4-x^2}}$
3. The minimum surface area of glass occurs when  $A(x)$  is maximal, at  $x = \sqrt{2}$ .  
The minimal surface is  $4^2 - 4A(\sqrt{2}) = 12 \text{ in}^2$ .

### Corrigé D2-72

1. The external and base surface is given by the formula:  $\pi r^2 + 2\pi r h$ . The equation  $\pi r^2 + 2\pi r h = 48\pi$  gives  $h = (48 - r^2)/(2r)$ . The volume is given by  $V = \pi r^2 h$ . Using the expression of  $h$  found previously we obtain  $V = 24\pi r - \pi/2 r^3$
2.
  - a. The derivative is given by  $dV/dr = 24\pi - 3\pi/2 r^2$
  - b. The stationary points are the solutions of the equation  $dV/dr = 0$  which gives  $r = 4$  or  $r = -4$ . Only the positive solution is retained as the measure of the radius has to be positive.
3. The study of the sign of the derivative allow Liam to state that the volume is maximum when  $r = 4$ . He can compute the value of  $h$  using the formula at question 1 obtaining  $h = 4$ .

### Corrigé D2-81

1) Surface area :  $S = 2x \times x \times 2 + h \times x \times 2 + h \times 2x \times 2$   
 $S = 4x^2 + 6hx$

2)  $S = 300$  donc  $300 = 4x^2 + 6hx$

Il vient :  $h = \frac{300 - 4x^2}{6x} = \frac{150 - 2x^2}{3x}$

3) Volume :  $V = 2x^2 \times h$

$$V = \frac{300x - 4x^3}{3}$$

$$V = 100x - \frac{4}{3}x^3$$

$$4) V'(x) = 100 - 4x^2 \text{ donc } V'(x) = 0 \Leftrightarrow x^2 = 25 \\ \Leftrightarrow x = \pm 5$$

Or  $x$  est une dimension donc  $x > 0$

$x$	0	5	$+\infty$
$V'(x)$	+	0	-
$V$			

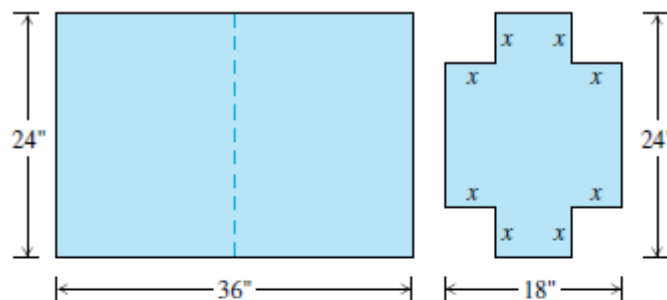
Le volume max est atteint pour  $x = 5$  cm et il vaut  $\frac{1000}{3}$  cm<sup>3</sup>, soit environ 333,33 cm<sup>3</sup>.

## Corrigé D2-82

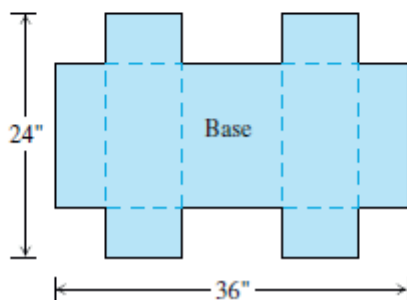
### Designing a Suitcase

A 24- by 36-in. sheet of cardboard is folded in half to form a 24- by 18-in. rectangle as shown in the figure.

Then four congruent squares of side length  $x$  are cut from the corners of the folded rectangle. The sheet is unfolded, and the six tabs are folded up to form a box with sides and a lid.



The sheet is then unfolded.



1. Write down a formula  $V(x)$  for the volume of the box.

$$V(x) = \text{Base} * \text{height} = (2x * (24 - 2x)) * (18 - 2x) = (48x - 4x^2)(18 - 2x) \\ = 864x - 96x^2 - 72x^2 + 8x^3 = 8x^3 - 168x^2 + 864x$$

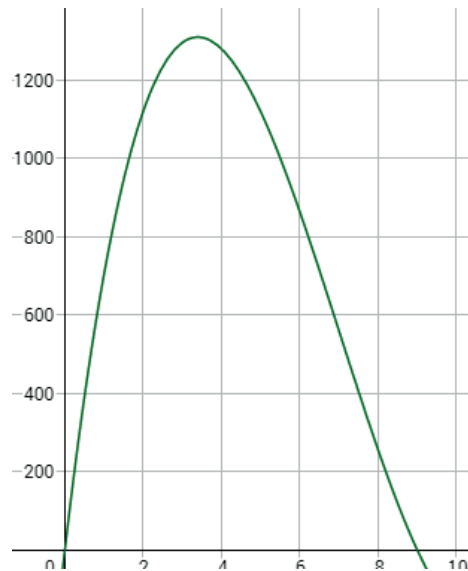
2. Check that  $V(x) = 8x(x^2 - 21x + 108)$ .

$$8x(x^2 - 21x + 108) = 8x^3 - 168x^2 + 864x$$

3. Find the domain of  $V$  for the problem situation.

$$0 < x < 9 \text{ so } D_V = ]0; 9[$$

4. graph  $V$  over this domain.



5. Use a your graph to find an approximate value of the maximum volume and the value of  $x$  that gives it. **The maximum volume seems to be reached at  $x = 3.5$  and its value is approximately  $1300 \text{ in}^3$ .**

6. Confirm your result from the previous question by a calculation. (hint : differentiate  $V$ )

$$V'(x) = 24x^2 - 336x + 864$$

$$V'(x) = 0$$

$$24x^2 - 336x + 864 = 0$$

$$\Delta = (-336)^2 - 4 * 24 * 864 = 29952 > 0$$

$$x_1 = \frac{336 - 48\sqrt{13}}{48} \approx 3,4 \text{ and } x_2 = \frac{336 + 48\sqrt{13}}{48} \approx 10,6 > 9$$

So on  $]0; x_1]$ ,  $V'(x) \geq 0 \rightarrow V$  is increasing

on  $[x_1; 9]$   $V'(x) \leq 0 \rightarrow V$  is decreasing

**So its maximum value is reached at  $x_1$  and its value is  $V(x_1) \approx 1310 \text{ in}^3$**

7. Find a value of  $x$  that yields a volume of  $1100 \text{ in}^3$ .

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**Domaine 3 – SEQUENCES – corrigés**

**CORRIGE D3-52**

1.  $u_1 = 2000 \times 1.05 = 2100$   $u_2 = (2100 - 200) \times 1.05 = 1995$
2.  $u_{n+1} = (u_n - 200) \times 1.05 = 1.05 u_n - 210$
3.
  - a.  $V_{n+1} = u_{n+1} - 4200 = u_n \times 1.05 - 210 - 4200 = (u_n - 4200) \times 1.05 = 1.05 \times V_n$   
 $r = 1.05$  and  $V_1 = -2100$
  - b.  $V_n = -2100 \times 1.05^{n-1}$  and  $u_n = -2100 \times 1.05^{n-1} + 4200$
4.  $u_{15} \approx \text{£}42$  and  $u_{16} \approx \text{£} - 165$
5. ...

**Corrigé D3-62**

1.  $P_{n+1} = 1.02 P_n$  and  $P_1=8$   
 $P_n = 8 \times 1.02^{n-1}$
2.  $F_{n+1} = F_n + 0.4$   
 $F_n = 10 + (n - 1) \times 0.4 = 0.4n - 9.6$
3. 1850 corresponds to  $n = 50$ .  
 $P_{50} \approx 21.110$  millions  
 $F_{50} = 29.6$  millions
4. In year 1888 the feedable population will be less than the total population.

$n = 87$	$P_{87} \approx 43.924$	$F_{87} = 44.4$
$n = 88$	$P_{88} \approx 44.803$	$F_{88} = 44.8$

**Corrigé D3 - 63**

1.  $u_4 = 22$
2.
  - a. There are  $n$  dots on the side of the  $n^{\text{th}}$  pentagon.
  - b. To get  $u_{n+1}$ , you start with  $u_n$ , add three times the side of the pentagon  $P_{n+1}$  (you get  $3(n+1)$ ); but 2 dots have been counted twice; so you get finally :  
 $u_{n+1} = u_n + 3(n+1) - 2 = u_n + 3n + 1$

3.  $v_n = 3n + 1$

a. The sequence  $(v_n)$  is arithmetic with common difference 3 and first term  $v_1 = 4$ .

b.  $v_1 + v_2 + v_3 + \dots + v_{n-1} = (n-1) \times \frac{(4 + 3(n-1) + 1)}{2} = \frac{(n-1)(3n+2)}{2}$

$v_1 + v_2 + v_3 + \dots + v_{n-1} = u_2 - u_1 + u_3 - u_2 + \dots + u_n - u_{n-1} = u_n - u_1$

c.  $u_n = u_1 + \frac{(n-1)(3n+2)}{2} = 1 + \frac{(n-1)(3n+2)}{2}$

4.  $u_{10} = 145$

Pentagon : 5 diagonals can be drawn in total.

Hexagon : 9 diagonals can be drawn in total.

a) *How many diagonals in total can be drawn in the  $n$ th shape ?*

$\frac{1}{2}(n-1)(n+2)$  diagonals can be drawn in the  $n$ th shape.

2. *An inscribed triangle is a triangle that has all of its vertices common with the given polygon. How many inscribed triangles in total can be drawn in each of the shapes above?*

Triangle : 1 triangles can be drawn in total.

Square : 4 triangles can be drawn in total.

Pentagon : 10 triangles can be drawn in total.

Hexagon : 20 triangles can be drawn in total.

Regular polygon with  $n$  sides : " $n$  choose 3" triangles can be drawn in total.

### Corrigé D3 - 65

#### Business A

1. (a)  $a_1 = 50$ ,  $a_2 = 60$ ,  $a_3 = 70$ .  $(a_n)$  is an A.P. with first term  $a_1 = 50$  and common difference  $d = 10$ .

(b)  $a_n = a_1 + (n-1)d = 40 + 10n$ .

2. (a)  $A_1 = a_1 = 50$ ;  $A_2 = a_1 + a_2 = 110$ ;  $A_3 = a_1 + a_2 + a_3 = 180$ .  $A_n$  is the sum of the first  $n$  terms of an A.P.

(b)  $A_n = \frac{(a_1 + a_n)}{2} \times n = \frac{50 + 40 + 10n}{2} \times n = n(5n + 45)$ .

#### Business B

Sequence  $(b_n)$  is a G.P. with first term  $b_1 = 40$  and common ratio  $r = 1.1$ .

Hence,  $b_n = 40 \times 1.1^{n-1}$ .

$A_n$  is the sum of the first  $n$  terms of a G.P.

Hence,  $A_n = 40(1 + 1.1 + 1.1^2 + \dots + 1.1^{n-1}) = 40 \frac{1.1^n - 1}{1.1 - 1} = 400(1.1^n - 1)$ .

#### Business A or Business B?

$A_{43} = 11180$ , while  $B_{43} = 23696$  to the nearest pound.

For a 43-metres-long well, the offer A is the best.

Note: offer A becomes best as soon as  $n$  is greater than 28.

### Corrigé D3 - 71

1) In step 4, we should add 4 hexagons.

In step 1, the extra number of sticks is 6; in step 2, the extra number of sticks is 9; in step 3, the extra number of sticks is 12. In step 4, the extra number of sticks should be 15.

2) Copy and complete the table below:

Step $n$	1	2	3	4
Number of hexagons on line $n$ : $h_n$	1	2	3	4
Total number of hexagon at step $n$ : $H_n$	1	3 (1+2)	6 (1+2+3)	10 (1+2+3+4)
Extra number of sticks in line $n$ : $T_n$	6	9	12 (9+3)	15 (12+3)
Total number of sticks used at step $n$ : $S_n$	6	15 (6+9)	27 (15+12)	42 (27+15)

1)  $H_n = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

2)  $T_{n+1} = T_n + 3$  : ( $T_n$ ) is an arithmetic progression with common difference 3. The first term is  $T_1 = 6$ .

3) Therefore, for any integer  $n$ ,  $T_n = 6 + 3(n - 1)$  and  $S_{n+1} = S_n + T_{n+1} = S_n + 6 + 3n$ .

4) If  $n = 3$  (be careful, for  $n = 2$ , we can't eliminate the third formula):

$$2 \times 3^2 + 4 \times 3 = 30 \neq 27, \text{ the first formula isn't working.}$$

$$\frac{3}{2} \times 3^2 + \frac{9}{2} \times 3 = 27 : \text{ the second is working}$$

$$\frac{5}{2} \times 3^2 + \frac{3}{2} \times 3 + 2 = 29 \neq 27, \text{ the third formula isn't working.}$$

Therefore, the formula is  $S_n = \frac{3}{2}n^2 + \frac{9}{2}n$ .

Another way to find this formula is:  $S_n = 6 + (6 + 3) + (6 + 2 \times 3) + \dots + (6 + (n - 1) \times 3)$

$$= 6 \times n + 3(1 + 2 + \dots + (n - 1))$$

$$= 6 \times n + 3 \times \frac{n(n-1)}{2}$$

$$= \frac{3}{2}n^2 + \frac{9}{2}n$$

5) By trial and improvement. Or :

We want to find the value of  $n$  such as  $S_n = 105$ :

$$\frac{3}{2}n^2 + \frac{9}{2}n = 105 \text{ then } 3n^2 + 9n - 210 = 0: \text{ it's a quadratic equation with } \Delta = 2601 = 51^2$$

Therefore the equation has two solutions:  $n_1 = \frac{-9-51}{2} = -30$  and  $n_2 = \frac{-9+51}{2} = 21$ .

$n$  is a positive number, thus  $n = 21$  : there are 21 rows in the final step.

The number of hexagons is:  $H_{21} = \frac{21 \times 22}{2} = 231$ .

### Corrigé D3 - 72

In your new 'get-fit' program, you plan to jog 1,500 metres on the first night and then increase this distance by 250 metres each subsequent night.

1. First, focus on the distance jogged each night, and complete the table.

Night number	1	2	3	4
Distance jogged (m)	<b>1,500</b>	<b>1,750</b>	<b>2,000</b>	<b>2,250</b>

2. If you continue the pattern, write down an expression for  $D_n$ , the distance jogged on the  $n$ th night.

$$D_n = 1,500 + (n - 1) \times 250 = 1,250 + 250n$$

3. How far will you jog :

a) On the 7th night ?

$$D_7 = 1,500 + 6 \times 250 = 3,000$$

b) On the 12th night ?

$$D_{12} = 1,500 + 12 \times 250 = 4,500$$

4. Now focus on the total distance jogged over several nights, and complete the table.

Night number	1	2	3	4
Total distance jogged (m)	<b>1,500</b>	<b>3,250</b>	<b>5,250</b>	<b>7,500</b>

5. Write down an expression for  $S_n$ , the total distance jogged after  $n$  nights.

$$S_n = n \times \frac{1,500 + 1250 + 250n}{2} = 1,375n + 125n^2$$

6. Determine the total distance you expect to jog after ten nights.

$$S_{10} = 1,375 \times 10 + 125 \times 10^2 = 13,750 + 12,500 = 26,250 \text{ m}$$

7. Determine the number of nights you will need to stick to your program to ensure that you jog a total of more than 50 km.

$$S_{16} = 54 \text{ km}$$

So, it will take 16 nights.

### Corrigé D3-81

The first day, the Ceryneian Hind runs 50 kilometres and since it is chased, each day it will run 10 more kilometres than the day before.

The first day, Heracles will run 40 kilometres and since he really needs to catch the hind, each day he will run a 5% longer distance than the day before.

Let denote by  $a_n$  the distance run by the hind on the  $n$ -th day and by  $b_n$  the distance run by Heracles on the  $n$ -th day.

$(a_n)$  is an arithmetic sequence with common difference equal to 10 and  $(b_n)$  is a geometric sequence with common ratio equal to 1.05.

1/ Calculate the distance run by the Ceryneian Hind and Heracles the second day.

$$a_2 = a_1 + 10 = 50 + 10 = 60$$

The Ceryneian Hind will run 60 km the second day.

$$b_2 = b_1 \cdot 1.05 = 40 \cdot 1.05 = 42$$

Heracles will run 42 km the second day.

2/ Calculate the distance to the nearest kilometre run by the Ceryneian Hind and Heracles the tenth day.

$$a_{10} = a_1 + 9 \cdot 10 = 50 + 90 = 140$$

The Ceryneian Hind will run 140 km the tenth day.

$$b_{10} = b_1 \cdot 1.05^9 = 40 \cdot 1.05^9 = 62$$

Heracles will run 62 km the tenth day.

3/ Find out the first day Heracles runs a longer distance than the Ceryneian Hind.

$$a_{56} = 600 \text{ and } a_{57} = 610$$

$$b_{56} = 585 \text{ and } a_{57} = 615$$

Starting from the 57<sup>th</sup> day, Heracles will run daily a longer distance than the Ceryneian Hind.

4/ How many days does Heracles need to catch up with the Ceryneian Hind ? Calculate the total distance to the nearest kilometre run by the animal and the hero before he manages to capture it.

We have to calculate the arithmetic and geometric series :

we calculate the total distance run by the Ceryneian Hind the 76<sup>th</sup> day :

$$S = a_1 + a_2 + \dots + a_{76} = 76 \times \frac{a_1 + a_{76}}{2} = 76 \times \frac{a_1 + a_1 + 75 \times 10}{2} = 76 \times \frac{50 + 50 + 750}{2} = 76 \times \frac{850}{2} = 76 \times 425$$

$$S = 32300$$

and since  $a_{77} = a_1 + 76 \times 10 = 50 + 76 \times 10 = 810$ , the 77<sup>th</sup> day, the hind will have run a total of 33 110 km.

We calculate the total distance run by Heracles the 76<sup>th</sup> day :

$$S = b_1 + b_2 + \dots + b_{76} = b_1 \times \frac{1 - 1.05^{76}}{1 - 1.05} = 40 \times \frac{1 - 1.05^{76}}{-0.05} = (-800) \times (1 - 1.05^{76}) \simeq 31819$$

and since  $b_{77} = b_1 \times 1.05^{76} = 40 \times 1.05^{76} = 1631$ , the 77<sup>th</sup> day, Heracles will have run a total of 33 450 km.

It means that Heracles captures the hind on the 77<sup>th</sup> day.

### Corrigé D3-82

- Let  $a_n$  be the number of followers on the  $n$ th day. Then, on day  $n + 1$ , Albert gets 10% more minus 5, thus,  $a_{n+1} = 1.1a_n - 5$ .
- If  $a_1 = 50$ , then  $a_2 = 1.1 \times 50 - 5 = 50$ ,  $a_3 = 50 \dots$  the sequence is always 50. Thus  $a_{30} = 50$  and  $a_{365} = 50$ .
- If  $a_1 = 50$ , the sequence is equal to 50 for every integer  $n$ .
- If  $a_1 < 50$ , the sequence decreases to 0.
- 

a.  $a_{n+1} = 1.1 a_n - 5 \Leftrightarrow u_{n+1} - 50 = 1.1(u_n - 50) - 5 \Leftrightarrow u_{n+1} = 1.1u_n$ . Therefore  $(u_n)$  is a geometric progression, with common ratio  $q=1.1$  and  $u_1 = a_1 - 50 = 51 - 50 = 1$

b. Thus  $u_n = 1.1^n \times u_1 = 1.1^n$ . And so  $a_n = 1.1^n + 50$

c. By trial and improvement. Or :

Let's solve  $1.1^n + 50 = 300 \times 10^6 \Leftrightarrow n = \ln(300 \times 10^6 - 50) / \ln(1.1) = 204,8$ . Thus Albert will be followed by the entire network after 205 days.

### Corrigé D3-83

- $J_2 = 500 + 150 = 650$  and  $J_3 = 650 + 150 = 800$ .

John will have saved £650 after 1 month and £800 after 2 months.

$M_2 = 5 \times 2 = 10$  and  $m_3 = 10 \times 2 = 20$ .

Mary will have got £10 after 1 month and £20 after 2.



2. John adds the same amount (i.e. £150) of money every month. Consequently,  $(j_n)$  is an arithmetic sequence with common difference 150.

Mary multiplies the amount of money she already got by 2 each and every month ; thus,  $(m_n)$  is a geometric sequence with common ratio 2.

3.  $j_n = 500 + 150n$

4.  $m_n = 5 \times 2^n$ .

5. 1 year = 12 months, thus  $j_{12} = 500 + 150 \times 12 = 2,300$  and  $m_{12} = 5 \times 2^{12} = 20,480$ .

6. Using the function table of a calculator,

If  $n = 8$  then  $j_8 = 1,700$  and  $m_8 = 1,280$ .

If  $n = 9$  then  $j_9 = 1,850$  and  $m_9 = 2,580$

Mary will become richer than John after 9 months.

### CORRIGE D3 - 84

1.  $u_n$  is neither a GP nor an AP.

2. Yes

- 3.

a.  $U_n + V_n = 3(2^n + 4^n)$  is not a GP

b.  $U_n + V_n = 7 \times 3^n$  is a GP

c. HARD : be kind. The sum of two GP s is a GP IFF the common ratios are the same.

4. HARD : be kind !

$$b_n = a_n - 2 \text{ so } b_{n+1} = a_{n+1} - 2 = 3a_n - 4 - 2 = 3a_n - 6 = 3(a_n - 2) = 3b_n$$

## **Domain 4 – Statistics and Probability**

### **Corrigé de statistics D4 – 41**

1) I would recommend Route 1 because the mean is lower ; even though the standard deviation is a bit higher so that travel times vary more than with route 2. Nevertheless over a period of time, route 1 would be less time consuming.

or

I would recommend Route 2 even though the mean is a bit higher, because the standard deviation is lower so travel time would be more constant and reliable.

2) (a) strong positive correlation

The older the car, the higher the mileage or the higher the number of kilometres travelled.

(b) (i) It is a 2 years old car which has travelled 40,000 kilometres.

(ii) This car is owned by someone who travels a lot, maybe for professional reasons, it could be a sales representative ; or for personal reasons, it could be someone who works in a part of the country but has a friend in another part of the country.

(c) (i)  $y=ax+b$  with  $a=8221$  and  $b=4019$

(ii) We have to solve an equation :  $35000=8221x+4019$  then  $x=30981/8221 = 3.8$  years

(=3years and 9.6months)

(iii) 6 is close to the values of  $x$ . We can use the best fit line :  $y = 8221*6+4019 = 53345$  km

10 is far from the values of  $x$ , we can't use the best fit line (eg the car could have been broken down).

## Corrigé de statistics D4 – 42

- a)  $\bar{x} = \frac{7+6+\dots+7}{8} = 6.875$  and  $\bar{y} = \frac{113+67+\dots+76}{8} = 83.5$   
 b) x axis from 3 to 10 ; y axis from 32 to 120 ; 8 points to plot  
 c) weak positive correlation  
 d) .  
 e)  $y=10.74x+9.68$  to 2dp. To draw it, two points. For instance : (0,9.68) and  $(\bar{x}, \bar{y})$   
 f)  $3 \leq 8 \leq 10$  we can use the best fit line (graphically or by calculation)  
 $10.74 * 8 + 9.68 = 95.6 \approx 96$  major thefts.  
 g)  $32 \leq 40 \leq 120$  we can use the best fit line (graphically or by calculation)

$$10.74x + 9.68 = 40 \text{ then } x = \frac{30.32}{10.74} = 2.82\%$$

## Corrigé de statistics D4 – 51

### Questions

- Yes - no
- Compute the mean, median, interquartile range for both distributions.

	Ap1		Ap2	
$\bar{r}$	63.75		63.6	
median	67.5		64.5	
$\xi$	English way 58.5	French way 58	English way 53.5	French way 53
$\xi$	71.5	71	72.5	72
Interquartile range	13	13	19	19

- Find the modal values, explain how to retrieve them from the stem-and-leaf plot.

1 mode for Ap1 : 71

2 modes for Ap2 : 54 and 72

- From the measurements that you have computed, would you rather hire Ap1 (applicant #1) or Ap2 (applicant #2) ? Explain the reasons for your choice.

Ap1 gave a rough job, Ap2 has a better spread about the median.

- Draw a histogram for the grades given by Ap2. Describe the relationship between the stem-and-leaf plot and the histogram.

Rotate the stemplot 90° anticlockwise and you obtain a histogram with tens as class intervals, all this because all leaves are the same size and same distance one from the next.

Or : usual way

## Corrigé de statistics D4 – 52

### Exercise 1

The annual salaries of the employees in a small company are listed below in ascending order :

€ 20,000      €22,000      €24,000      €25,000      €30,000      €105,000

The mean salary is €37,667 rounded to the nearest whole number.

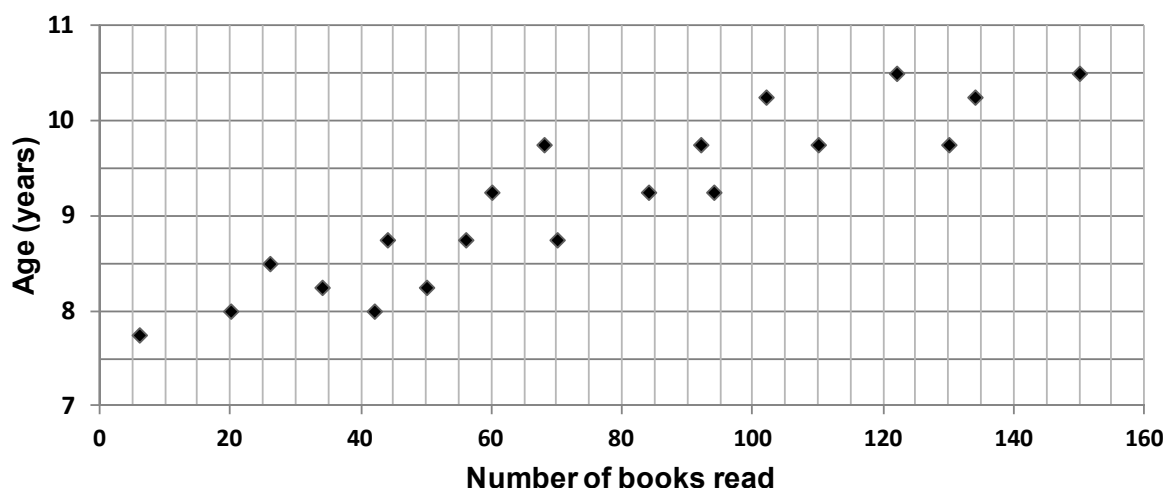
(a) (i) the median salary is  $(24,000 + 25,000) \div 2 = €24,500$

(ii) All salaries are different, there is not any salary appearing even twice so there is no mode.

(b) Since only one salary is greater than the mean, and that 5 salaries are less than the mean, the mean is not a good representative of the typical salary. The median is the best choice here, as it is not affected by the highest salary.

### Exercise 2

This scatter graph shows the number of books read by some children and the ages of these children.



(i) We count 6 children who have read more than 100 books.

(ii) The child who has read 50 books is  $8\frac{1}{4}$  years old.

(iii) The graph shows that as the number of books read increases, so does the age of the child. Or the older the child, the more books he or she has read. There is a strong positive correlation between the age of the child and the number of books the child has read.

(iv) The new child has read 170 books. We assume we can use the line of best fit to estimate how old this child is. We take  $x = 170$  and the equation yields  $y = 11.1$  so the child is slightly older than 11.

## Corrigé de statistics D4 – 53

### Question 1

route	mean	median	mode	range
<b>A</b>	21.95	22	22	16
<b>B</b>	<b>21.6</b>	<b>21</b>	<b>24</b>	<b>11</b>

Students could mention other relevant factors, such as preferring to walk or take the bus, perhaps differently depending on the weather.

### Question 2

Statement	Mean	Median	Mode	Range
It's equal to zero.	<i>Possible if all the values are zero or some are negative so that their total comes to zero.</i>	<i>Possible; e.g.</i> -10, -1, 0, 2, 3	<i>Possible; e.g.</i> 0, 0, 1, 2, 3	<i>Possible if all the values are equal.</i>
It's the highest value.	<i>Possible only if all the values are the same.</i>	<i>Possible only if all the values are the same.</i>	<i>Possible; e.g.</i> 1, 2, 3, 4, 4	<i>Possible if the lowest value is zero.</i>
It's less than any of the values.	<i>Impossible, because the mean represents equal shares of the total amount.</i>	<i>Impossible, because the "middle" value can't be less than any of them.</i>	<i>Impossible, because the mode must be an actual value.</i>	<i>Possible if the lowest value is negative; e.g.</i> -2, 3, 4, 5

## Corrigé de statistics D4 – 54

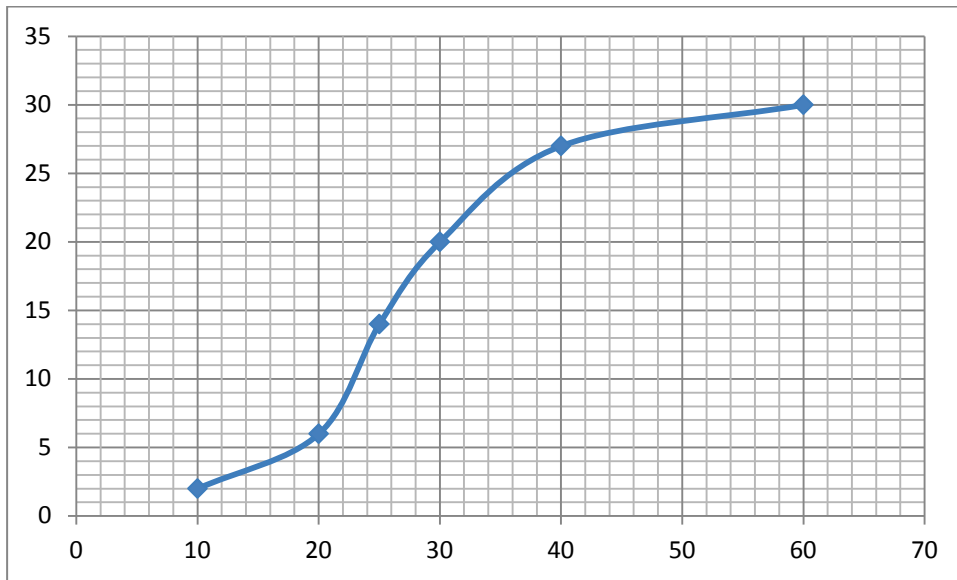
Answer:

- 1)  $\bar{h} = \frac{64+\dots+63}{9} \approx 64.44 \text{ inches} \approx 64.44 \times 2.54 \text{ cm} \approx 164 \text{ cm} \approx 1 \text{ m} 64$
- 2)  $\bar{w} = \frac{132+\dots+130}{9} \approx 135.88 \text{ pounds} \approx 135.88 \times 0.454 \text{ kg} \approx 61.7 \text{ kg}$
- 3) Two axes: on the x-axis the height from at least 55 to 75; on the y-axis, the weight from at least 124 to 150, we plot 9 points whose x-coordinates are the heights and whose y-coordinates are the weights. They seem to be on the same straight line so yes
- 4) It seems to be a strong positive correlation since points on the same line and line increasing  
Besides value of the correlation coefficient:  $r \approx 0.99$  very close to 1
- 5) On accepte la réponse à main levée.  
Sinon :  $h = 1.36w + 48.07$ . The line has to pass through the mid-point  $(\bar{h}, \bar{w}) = (64.44, 135.88)$   
*we can find the coordinates of another point (55, 122.87)*
- 6) You could either use the equation or the graph:  
(i) 137.83 pounds                      (ii) 101.11 pounds
- 7) 39 inches is too far from the existing data

## Corrigé de statistics D4 – 55

1) a..

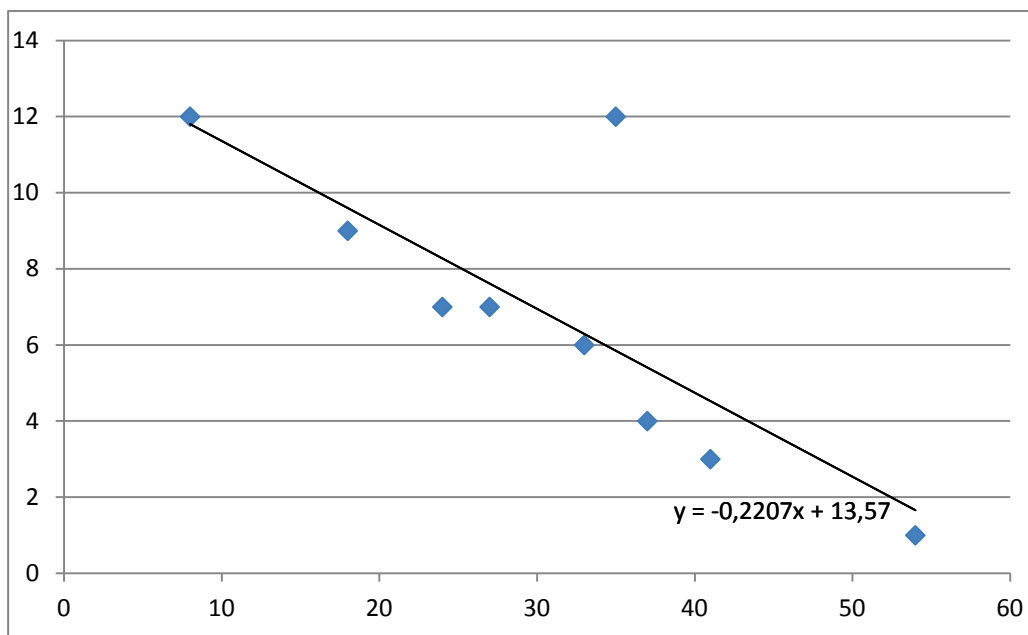
Time spent (in minutes)	Less than 10	10-20	20-30	30-40	40-50	50-60
Number of pupils	2	4	8	6	7	3
Less than cumulative frequency	2	6	14	20	27	30



Median approx. 26 minutes, LQ approx. 21 minutes and UQ approx. 33

1) b.  $UQ - LQ = 12$

2) a.



2) b. A strong negative linear correlation is shown except for an outlier (35,12).

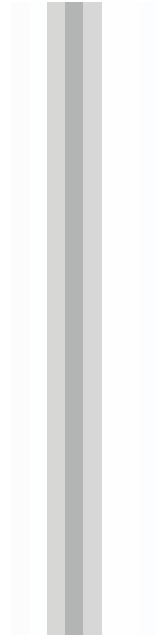
2) c. Equation of the line of best fit without the outlier  $y = -0.24x + 13.42$  and with the outlier  $y = -0.22x + 13.57$

2) d. About 8 mistakes thanks to the line of best fit.

# Corrigé de statistics D4 – 61

## Part A

- 1) This is a continuous quantitative data.
- 2) We have to draw a cumulative frequency graph.



On the x-axis, we plot the marks graduated from 0 to 20, and on the y-axis, the cumulative frequencies from 0 to 30.

Points : (0,0), (5,4), (10,12), (15,24) and (20,30). Then we joint hem with a smooth curve.

The median is the x-coordinate of the point of the curve whose y-coordinate is  $30/2$  : 15. Then the median is 11.

- 3) Half of the students obtained a mark less than or equal to 11 and half of the students obtained a mark greater than or equal to 11.
- 4) IQR = Upper Quartile – Lower quartile.  
 $30 \cdot 3/4 = 22.5$ , the UQ is the x-coordinate of the point whose y-coordinate is 22.5 :  
UQ=14.5  
 $30/4 = 7.5$ , then LQ=7.5  
IQR = UQ – LQ =  $14.5 - 7.5 = 7$

## Part B

Median : measure of average, and as  $13 > 11$ , we can guess that Mrs Smith's students are better than Mr. Mat's.

IQR : measure of dispersion , and as  $4 < 7$ , we can guess that Mrs Smith's class is more homogeneous than Mr. Mat.

## Part C

- 1) strong positive correlation
- 2) Use of the equation given by the calculator or of the graph :  
Equation given by the calculator:  $y = 1.2x - 1.9$   
As  $9 < 13 < 16$ ,  $y = 1.2 \cdot 13 - 1.9 = 13.7$ . English mean predicted: 13.7
- 3) 2 is not between 9 and 17, we can't use the line of best fit...



## Corrigé de statistics D4-71

1) Quantitative continuous data

2) 1<sup>st</sup> class interval: freq density = 0.4 and width = 10 then freq = fd \* width = 4

class	40-50	50-55	55-60	60-65	65-70	70-80
freq	4	2	6	8	5	5
Cumulative freq	4	6	12	20	25	30

3) A cumulative frequency curve

We draw 2 axes: the weight on the x axis and the cumulative frequency on the y axis

We plot the points (40,0), (50,4), (55,6), (60,12), (65,20), (70,25) and (80,30)

Total freq / 2 = 15: the median is roughly equal to 62

4) Yes, def of the median

5) Strong positive correlation (the correlation coefficient is 0.87, close to 1, and the points are close to an increasing straight line)

The heavier the pupil is, the taller he is.

6) Equation found with the calculator with the points (71,160) (72,170), (74,180), (75,170), (76,190), (78,190):  $y = 4.1x - 128.1$

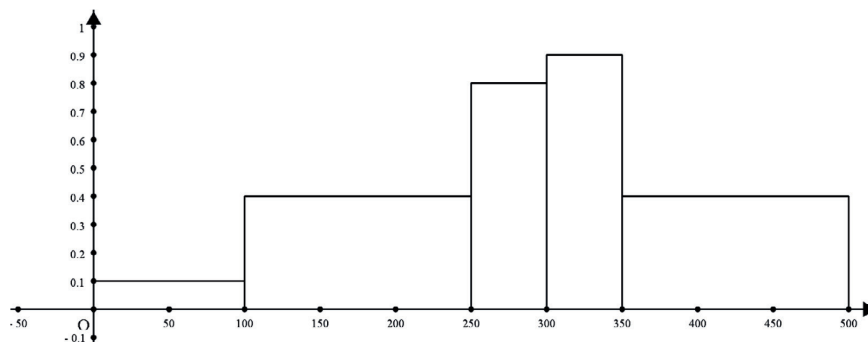
If  $x=73$  then  $y=171.2\text{cm}$ .

If the line is drawn by eye, we should find a similar value.

7) No, since 120 is too far from the existing data: the lowest height given is 160cm.

## Corrigé de statistics D4-72

The histogram below shows the price distribution of houses in an area of Manchester. Prices are given in thousands of pounds (to the nearest thousand).



Price £(x)000s	$0 \leq x < 100$	$100 \leq x < 250$	$250 \leq x < 300$	$300 \leq x < 350$	$350 \leq x < 500$
Frequency	$0.1 \times 100 = 10$	60	40	45	60
Cumulative frequency	10	70	110	155	215

1) We have to draw the bar for the last class 350-500. The corresponding frequency density is  $\frac{60}{500-350} = 0.4$ .

2)

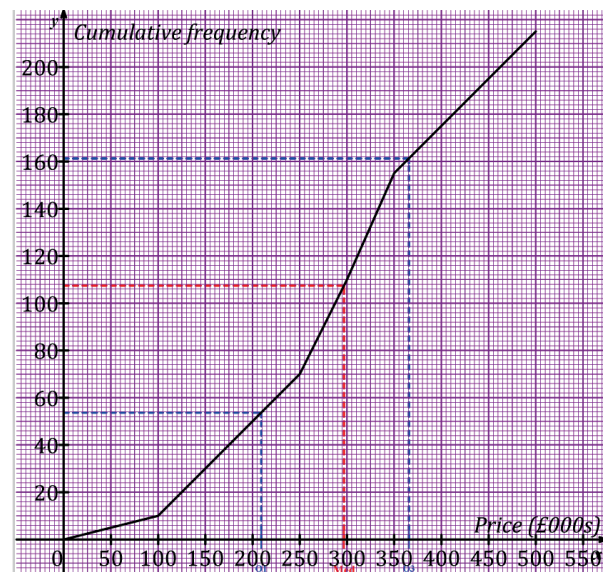
$$3) \bar{x} = \frac{50 \times 10 + 175 \times 60 + \dots + 425 \times 60}{215} \approx 288.953: \text{£ } 288,953.$$

4) 5)

6) median  $\approx 295$        $Q_1 \approx 210$        $Q_3 \approx 375$ .

7) Thanks to the graph, we read that 174 houses are sold Less than 400,000 pounds.

$215 - 174 = 41$ : 41 houses are sold more than £400,000.



### Corrigé D4-81

#### Answer:

1) a) two axes: x-axis: the age; y-axis: number of SMS sent

Plot the points whose coordinates are (age, number of sms)

1) b) the points seem to be collinear and seem to belong to the same increasing straight line, so positive correlation.

Calculator: correlation coefficient:  $r = 0.98$  close to 1 so strong positive correlation:

The older the child is, the more sms he sends

1) c) equation given by the calculator:  $y = 2.94x - 7.58$

It passes through  $(0, -7.58)$  and  $(\bar{x}, \bar{y}) = (9.8, 21.2)$ . (Or you draw it by eye but it has to pass through  $(\bar{x}, \bar{y})$ )

1) d)  $y = 20$ , so solving equation  $2.94x - 7.58 = 20$ , you find  $x \approx 9.4$ , so 9 years old

1) e) no since the age of 50 is too far from the existing data (between 7 and 15 years)

2) It's  $\bar{y} \approx 21$  sms per day (mean)

3) median: you put the values in increasing order: 10, 15, 16, 30, 35

And you take the middle value: 16

4)  $31/2=15.5$ . the median is the x-coordinate of the point the y-coordinate of which is 15.5. so the median is roughly 18.

It means: For half of the days of January, John took less than 18 photos and for half of the days of January, he took more than 18 photos.

### Corrigé D4-82

1. Use the ordered pairs given in the graph (0; 57.3) and (48; 48.7) to find a linear equation to estimate the winning time for the men's 100-m freestyle versus the year. Round the slope to 2 decimal places.

**Let  $y = ax + b$ , be the equation of the straight line.**

**The graph passes through the point (0; 57.3), so  $b = 57.3$ .**

$$a = \frac{57.3 - 48.7}{0 - 48} \approx -0.18$$

$$y = -0.18x + 57.3$$

2. Use the linear equation from question 1 to approximate the winning 100-m time for the year 1972, and compare it with actual winning time of 51.2 sec.

$$1972=1948+24 \text{ and } 1996=1948+48$$

$$\text{so } y = -0.18 \times 24 + 57.3 \approx 53 \text{ sec} > 48.7$$

3. Use the linear equation to approximate the winning time for the year 1988.

$$1988=1948+40, \text{ so } y = -0.18 \times 40 + 57.3 = 50.1 \text{ sec.}$$

4. What is the slope of the line and what does it mean in the context of this problem ?

**The slope is equal to  $-0.18$  and it measures how much the winning time for men 100m changes. In this case, the slope is negative so the winning time decreases throughout the years.**

5. Explain why the men's swimming times will never reach the  $x$  –intercept.

**The  $x$ -intercept give the moment when the winning time for men 100m is equal to 0 sec.**

**It is impossible because the time must at least greater than 0, unless you are superman.**

6. Do you think this linear trend will continue for the next 50 years, or will the men's swimming times begin to level off at some time in the future ? Explain your answer.

**It may stay constant at one moment, because we will maximise our winning time.**

### Corrigé D4-83

#### Exercise 1

- (a) Fill the gaps in this sentence:  
14 hours

- (b) Write a similar sentence, comparing groups A and B, using the lower quartile of group A. About three quarters of the teenagers in group A watched TV at least 12 hours that week, whereas about half of the teenagers in group B watched TV no more than 12 hours.
- (c) Work out the interquartile ranges of hours of TV in both groups.  
 IQ range in group A:  $17 - 12 = 5$  hours  
 IQ range in group B =  $14 - 9 = 5$  hours
- (d) Did the teenagers in group B spend more time watching TV than in group A?  
 No they did not. Even though a small percentage (less than 25 %) in group A watched very little TV that week, it would be quite correct to say that teenagers in group A spent more time watching TV.

### Exercise 2

(i) Ian : 81.2 & William : 80.2

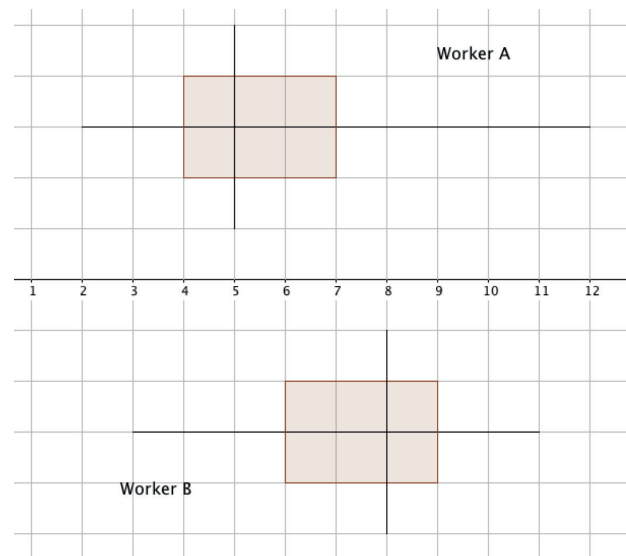
(ii)  $\sigma_{\text{Ian}} \approx 6$  and  $\sigma_{\text{William}} \approx 9$

Either one of them depending on whether the student thinks the regularity of the results is important or not.

## Corrigé D4-84 statistics

### Exercise 1

- For worker A's times :
  - the median is 5 since there are 10 times and that the 5th and 6th times are both 5 when the series is in ascending order.
  - the lower quartile is the 3rd time : 4, and the upper quartiles is the 8th time : 7
- For worker B's times :
  - the median is 8 since there are 10 times and that the 5th and 6th times are both 8 when the series is in ascending order.
  - the lower quartile is the 3rd time : 6, and the upper quartiles is the 8th time : 9
- see graph
- Worker B is more regular than worker A but seems globally slower. It seems that worker A is more efficient, so he's the one to employ.



### Exercise 2

- The mean weight of a brand A chocolate drop is  $\frac{60.3}{20} = 3.015$  grams.
- $\frac{20 \times 3.015 + 30 \times 2.95}{50} = 2.976$  so the mean of the weight of all 50 chocolate drops is 2.9 grams.

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**Domain 5 – Advanced Geometry – corrigés**

**Corrigé SUJET D5-41**

$$41 + 30 = 71^\circ$$

In right triangle ABH:  $\frac{x}{AH} = \tan 30 = \frac{1}{\sqrt{3}}$ , hence  $AH = x\sqrt{3}$

In right triangle BHC:  $\frac{x}{HC} = \tan 60 = \sqrt{3}$ , hence  $HC = \frac{x}{\sqrt{3}}$

Thus:  $AC = x\sqrt{3} + \frac{x}{\sqrt{3}} = \frac{4}{\sqrt{3}}x$

Area of the city:  $area(ABC) = \frac{AC \times x}{2} = \frac{2}{\sqrt{3}}x^2$ .

We are looking for  $x$  such that  $\frac{2}{\sqrt{3}}x^2 \geq 16,000,000$ .

To the nearest metre:  $x \geq 3723$ .

Take a perpendicular to the North direction... answer is  $360 - 19 = 341^\circ$

## Corrigé SUJET D5-45

a)  $\angle POA = \theta = \frac{360}{2 \times 5} = 36^\circ$

b) In the right-angled triangle POA,

$$\cos(\theta) = \frac{OP}{OA} = \frac{OP}{1} \quad \text{so} \quad OP = \cos(36) \approx 0,8$$

$$AB = 2AP = 2 \sin(36) \approx 1,2$$

c)  $Area AOB = \frac{(Base \times height)}{2} = \frac{(AB \times OP)}{2} = \frac{(1,2 \times 0,8)}{2} = 0,48$

d)  $\tan(\angle AOQ) = \frac{AQ}{OA} = AQ$

$$AQ = \tan(36) \approx 0,72$$

$$Area OQR = \frac{(Base \times height)}{2} = \frac{(QR \times AO)}{2} = \frac{(1,44 \times 1)}{2} = 0,72$$

e)  $5 \times 0,48 < \pi < 5 \times 0,72$

## Corrigé SUJET D5-52

- 1) Constraint on the number of matches gives that  $300x + 450y \leq 6000$  so by dividing both sides by 150 ... done.
- 2) Constraint on the numbers of days gives that. (  $5x + 3y \leq 60$  )
- 3) The wanted region is the quadrilateral OBCD.
- 4)  $T=50x+70y$  so by isolating  $y$ ... done.
- 5) Draw  $y = -5/7x+10$
- 6) Using lines parallel to  $\Delta$ , the maximum turnover is given by the coordinates of point C at the intersection of lines BC and AC. Coordinates can be found by reading the graph (6.6,8.9) or by solving simultaneous equations...

## Corrigé SUJET D5-61

A shop stocks only sofas and beds.

A sofa takes up  $3 \text{ m}^2$  of floor area and is worth £600. A bed takes up  $4 \text{ m}^2$  of floor area and is worth £300.

The shop has  $45 \text{ m}^2$  of floor space to stock.

The shop stocks at least 3 sofas and 2 beds at any one time.

The insurance policy will allow a total of only £6000 of stock to be in the shop at any one time.

The shop stocks  $x$  beds and  $y$  sofas.

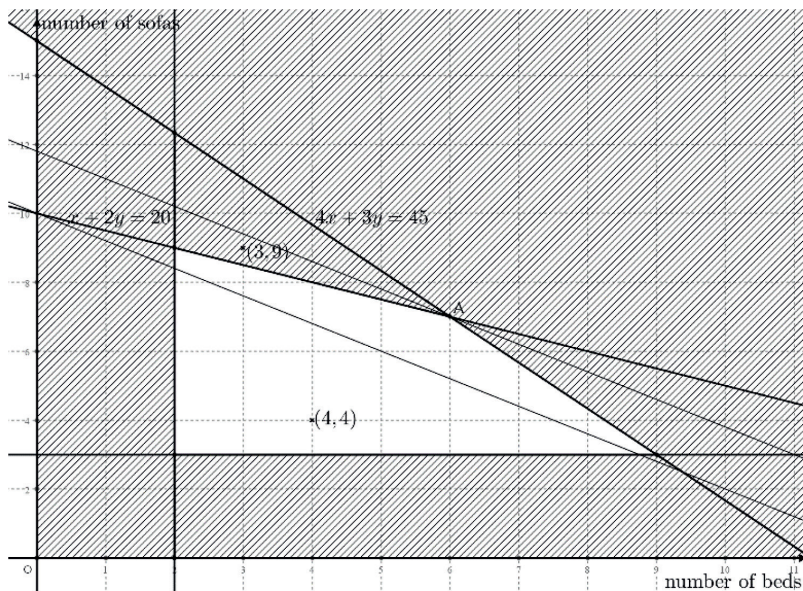
1) A bed is worth £300 and a sofa is worth £600.

If the shop stocks  $x$  beds and  $y$  sofas, its stock is worth  $300x + 600y$ . Because of the insurance policy, you should have:  $300x + 600y \leq 6000$ , which is the same as  $x + 2y \leq 20$ .

2)  $x \geq 2$ ,  $y \geq 3$  and  $4x + 3y \leq 45$ . (Furthermore,  $x$  and  $y$  are both integers)

3)  $4 \times 7 + 3 \times 6 = 46 > 45$  therefore the shop can't stock 7 beds and 6 sofas at any one time.

4) a)



b) tell if it's possible for the shop to stock at any one time:

(i) 4 beds and 4 sofas: yes

(ii) 3 beds and 9 sofas: no.

5) A is the intersection point of the lines  $4x + 3y = 45$  and  $x + 2y = 20$ . We'll work out the solution of the simultaneous equation: (6,7).

6) A bed is sold £800 and a sofa £1000.

a) An equation of D is  $800x + 1000y = 10,000$  which is the same as  $8x + 10y = 100$ . (0,10) and (5,6) are on this line.

b) If the shop sells 5 beds and 6 sofas (which is possible because (5,6) is in the wanted region), its revenue will be £10 000.

To find the maximum revenue the shop can make, we use the line D' passing through A and parallel to D.

An equation of D' is  $8x + 10y = 118$ . Therefore the maximum revenue is £11 800, if the shop sells 6 beds and 7 sofas



## Corrigé SUJET D5-81

Coordonnées des points dans le repère d'origine  $W$ .

$$\begin{array}{lll} W(0,0) & P(4\,500, -45) & M(92\,800, 0) \\ S(4\,500, 0) & R(88\,000, 0) & Q(88\,000, -40) \end{array}$$

$$(i) WP: -\frac{45}{4500} = -\frac{1}{100} \qquad PQ: \frac{5}{83500} = \frac{1}{16700} \qquad QM: \frac{40}{4800} = \frac{1}{120}$$

$$(ii) \text{ L'équation de } (PQ) \text{ est de la forme : } y = \frac{1}{16700}x + p$$

$$\text{Or } P \in (PQ) \text{ donc : } -45 = \frac{1}{16700} \times 4\,500 + p$$

$$\text{Finalement } (PQ) \text{ a pour équation : } y = \frac{1}{16700}x - \frac{7560}{167}$$

$$\text{Soit : } x - 16\,700y - 756\,000 = 0$$

$$(iii) * \text{ D'après le théorème de Pythagore on a : } WP^2 = WS^2 + SP^2$$

$$\text{D'où : } WP = \sqrt{4\,500^2 + 45^2} \approx 4\,500,22 \text{ m}$$

$$* \text{ D'après le théorème de Pythagore on a : } QM^2 = QR^2 + RM^2$$

$$\text{D'où : } QM = \sqrt{40^2 + 4\,800^2} \approx 4\,800,17 \text{ m}$$

$$* PQ = \sqrt{5^2 + 83\,500^2} \approx 83\,500 \text{ m}$$

$$* \text{ longueur totale : } 4\,500,22 + 4\,800,17 + 83\,500 \approx 92\,800 \text{ au mètre près.}$$

**Corrigé SUJET D5\_82**

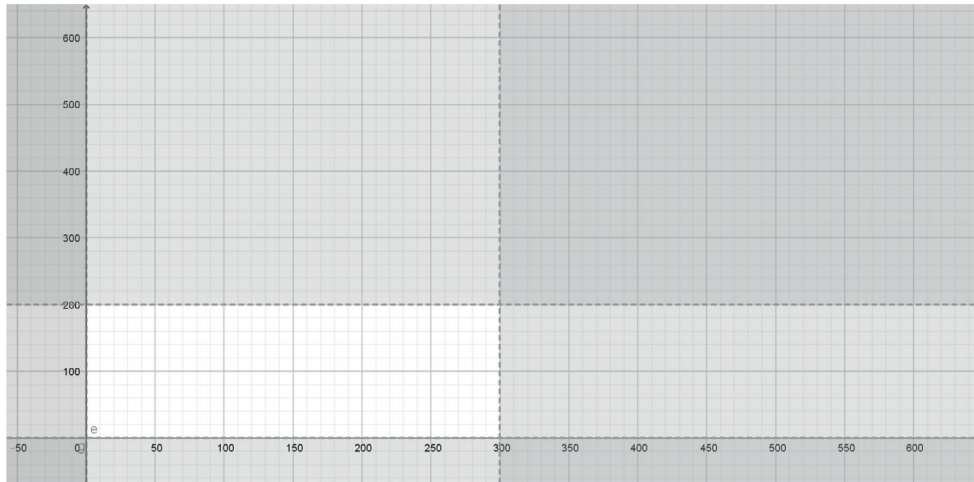
- 1-  $x + y \leq 1200$        $x \geq 200$        $y \geq 2x$
- 2- The wanted region is the triangle formed by the three lines.
- 3- No because the point is out of the wanted region or because 600 is not greater than 2 times 400.
- 4-  $P = 30x + 20y$
- 5- The previous line can be drawn for a random value of  $P$ . Then the line of maximum profit is the line parallel to the latter with the highest possible y-intercept and at least one point in the wanted region. The point of the wanted region that gives the maximum profit is (400,800) and the corresponding profit is 28000 pounds.

Corrigé SUJET D5\_83

- 1- Counting line from North at Heathrow, the bearing of A from Heathrow is  $040^\circ$
- 2- Many possible ways to find it using alternate interior angles or corresponding angles. The bearing of H from A is  $220^\circ$
- 3-  $\angle AHB = 90^\circ$  so  $\triangle AHB$  is right-angled at H so using Pythagoras we find  $AB = 86$  miles to the nearest mile.
- 4-
  - a. 45 minutes at 110mph is 90 miles
  - b. In triangle HTC,  $HT = TC = 90$  miles. Many ways to prove that  $\angle HTC = 120^\circ$   
Cosine rule gives  $HC^2 = 24300$  so  $HC = 156$  miles to the nearest mile.
  - c. Using the sine rule in HTC or the fact that HTC is an isosceles triangle we find that  $\angle THC = 30^\circ$  so the bearing of C from H is  $290^\circ$

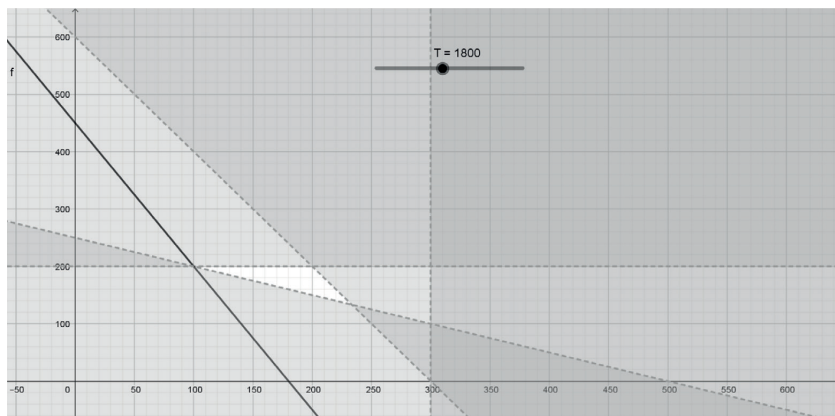
Corrigé SUJET D5\_84

- a. The factory cannot produce more than 200 yankee and 300 xtra pizzas.
- b. The white region is the accepted one (the number of pizzas cannot be negative).



2. *xtra pizza* : 5 mushrooms and 8 olives.      *yankee pizza* : 10 mushrooms and 4 olives.

- a. Number of mushrooms :  $5x + 10y \geq 2500$ .      Number of olives :  $8x + 4y \geq 2400$ .
- b. Mushrooms :  $y \geq 250 - 0.5x$ .      Olives :  $y \geq 600 - 2x$ .
- c. the feasible region : the white region.

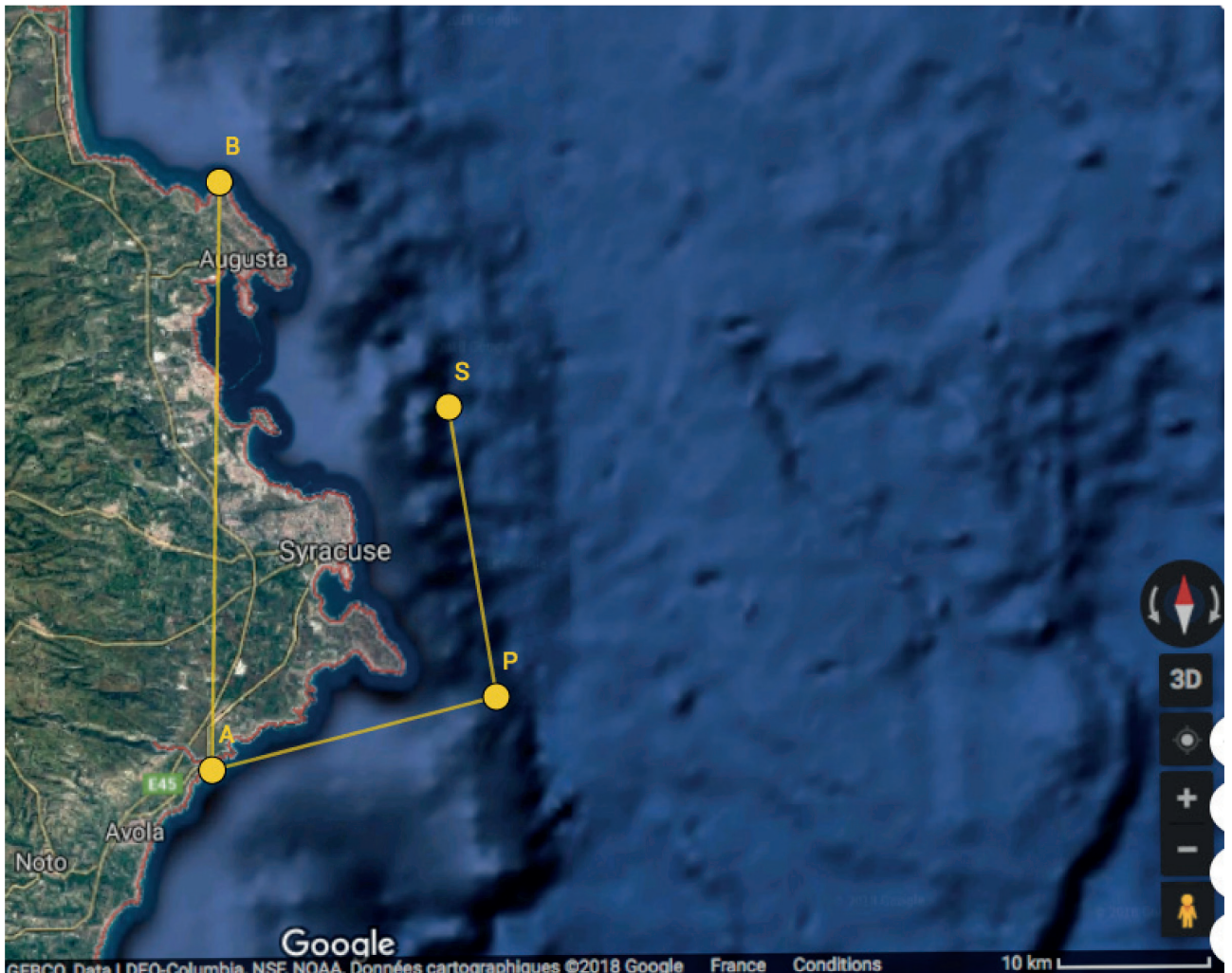


3. 10 minutes to produce each *xtra pizza* and 4 minutes to produce each *yankee pizza*.

- a.  $T = 10x + 4y$  then  $y = 2.5x - \frac{T}{4}$ .
- b. All the lines " $y=2.5x-T/4$ " are parallel because they have the same gradient 2.5. Bob needs a minimum time, so the y-intercept,  $T/4$ , must be minimized. We draw different parallel lines that cross the white region. The minimum time seems to be obtained when  $x = 100$  and  $y = 200$  then  $T = 10 \times 100 + 4 \times 200 = 1800$  minutes.

Corrigé SUJET D5\_85

1) Plot points P and S on the given diagram.



2) Let A be the origin of an orthonormal coordinate plane 1 km unit.

a. Give the coordinates of points A and B.

$$A(0; 0) \quad B(0; 40)$$

b. Work out the coordinates of points P and S. Give values to 1 d.p.

$$x_P = 20 \cdot \cos(15^\circ) \quad , \quad y_P = 20 \cdot \sin(15^\circ) \quad , \quad P(19,3; 5,2)$$

$$x_S = 20 \cdot \cos(15^\circ) - 20 \cdot \sin(10^\circ) \quad , \quad y_S = 20 \cdot \cos(10^\circ) + 20 \cdot \sin(15^\circ) \quad , \quad S(15,8; 24,9)$$

3) Compute the distance between S and B and give the result to 1 d.p.

$$SB^2 = (15,8 - 19,3)^2 + (24,9 - 5,2)^2 = 400,34$$

$$SB = 20 \text{ km}$$

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**PROBABILITY – Corrigé**

**Corrigé du sujet D7-43**

1. A cup is taken at random. It is then replaced and a second cup is taken at random.  
Using a tree diagram:

a. Probability that the two cups are decorated with Newton :  $p_1 = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$ .

b. Probability that the two cups are decorated with the same mathematician :  $p_2 = \left(\frac{3}{4}\right)^2 + \left(\frac{1}{4}\right)^2 = \frac{10}{16}$ .

c. Probability that at least one cup is decorated with Newton :  $p_3 = 1 - \left(\frac{1}{4}\right)^2 = \frac{15}{16}$ .

2.

a. 100 mugs are produced and on average 5% of them are defective.  
The probability that exactly 4 mugs are defective is :

$$P(X = 4) = \binom{100}{4} \times 0.05^4 \times 0.95^{96} = 0.178 \text{ to 3 d.p.}$$

b. 10 mugs are selected :

The probability that 2 or less mugs are defective is :

$$P(X = 0) + P(X = 1) + P(X = 2) = 0.95^{100} + 10 \times 0.05^1 \times 0.95^{99} + 45 \times 0.05^2 \times 0.95^{98} = 0.00977 \text{ to 6 d.p.}$$

So the probability that the batch will be rejected is a bit lower than 1% :  
the probability that the batch will be accepted is a bit higher than 99 %.

c. Complete inspection is more expensive, and can damage the products in some cases.

**Corrigé Sujet D7\_51**

1.  $p = \frac{\pi 10^2}{30^2} = \frac{\pi}{9} = 0.35 \text{ to 2 d.p.}$

2. (i)

(ii)  $P(S_1 \cap S_2) = 0.35 \times 0.4 = 0.14$

(iii)  $P(S_2) = P(S_1 \cap S_2) + P(\overline{S_1} \cap S_2) = 0.35 \times 0.4 + 0.65 \times 0.15 = 0.24 \text{ to 2 d.p.}$

(iv)  $P_{S_2}(S_1) = \frac{P(S_1 \cap S_2)}{P(S_2)} = \frac{0.14}{0.24} = 0.58$  to 2 d.p.

3. (i)  $P(X = 2) = \binom{6}{2} p^2 (1-p)^4 = 15 \times 0.35^2 \times 0.65^4 = 0.33$  to 2 d.p.

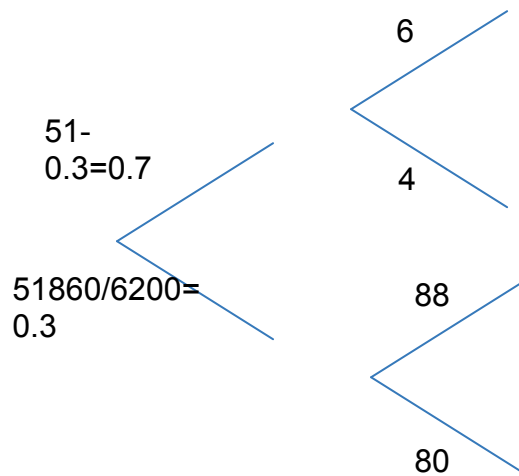
(ii)  $P(X \geq 1) = 1 - P(X = 0) = 1 - 0.65^6 = 0.92$

(iii) We should find  $n$  such that:  $1 - 0.65^n \geq 0.99$ , and we find  $n \geq 11$  (using the calculator or the logarithm function)

**Corrigé D7 - 61**

1.

a.



b.  $P(L) = 0.7 \times 0.6 + 0.3 \times 0.8 = 0.42 + 0.24 = 0.66 = \frac{33}{50}$

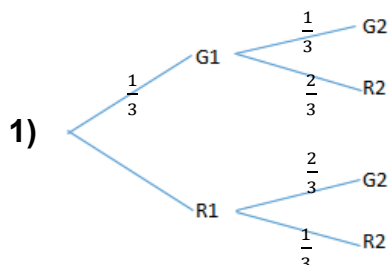
c.  $P(G|not L) = \frac{P(G \cap not L)}{P(not L)} = \frac{0.7 \times 0.4}{1 - 0.66} = \frac{0.28}{0.34} = \frac{14}{17}$

2. Let  $X$  be the number of containers that should be shipped to London amongst the 10 containers.  $X$  is a random variable that is distributed as a binomial distribution with parameters 10 and  $\frac{33}{50}$  because the containers are chosen independently.

a.  $P(X = 5) = 0.1434$  to 4 dp.

b.  $P(X \geq 2) = 1 - P(X = 0) - P(X = 1) = 0.9996$  to 4 dp.

**Corrigé D7 - 62**



$$\frac{2}{3}$$

- 2) a)  $P(G1 \cap G2) = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$   
 b)  $P(G1 \cap R2) + P(R1 \cap G2) = \frac{1}{3} \times \frac{2}{3} + \frac{2}{3} \times \frac{2}{3} = \frac{5}{9}$   
 c) Let  $H$  be the event "being held up at least once". Not  $H$  is the event: "not being held up at either set of lights". Therefore:  $P(H) = 1 - \frac{1}{9} = \frac{8}{9}$   
 d)  $P(G1 \cap R2) + P(R1 \cap R2) = \frac{2}{9} + \frac{2}{3} \times \frac{1}{3} = \frac{4}{9}$

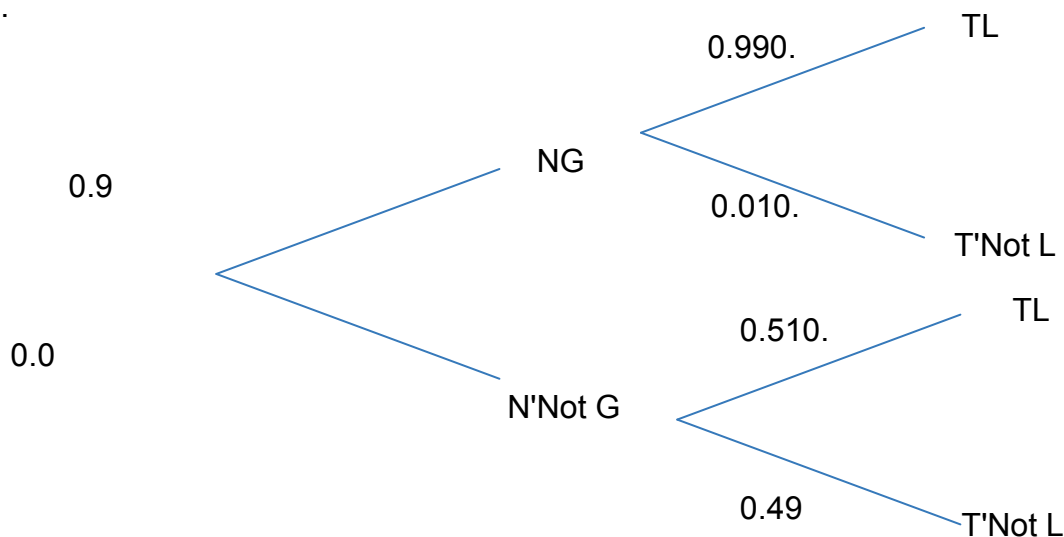
3)  $P(G1|R2) = \frac{P(G1 \cap R2)}{P(R2)} = \frac{2}{9} \times \frac{9}{4} = \frac{1}{2}$

4) The probability to have two green lights is  $\frac{1}{9}$ .

Therefore, I expect to get two green lights on  $90 \times \frac{1}{9} = 10$  journeys.

### Corrigé D7 - 63

1.



2.  $P(T \cap N) = 0.99 * 0.95 = 0.9405$

3.

- a.  $T' \cap N'$  is the event "the machine doesn't work well, and does not produce a good quality toy". Using the tree diagram, we find that  $P(T' \cap N') = 0.05 * 0.49 = 0.0245$   
 b. The probability that a toy is good quality is  $P(T) = 0.99 * 0.95 + 0.51 * 0.05 = 0.966$   
 So the CEO is wrong.

4. According to Bayes' rule :  $P(N|T) = \frac{P(T|N) \times P(N)}{P(T)} = \frac{0.99 \times 0.95}{0.966} \cong 0.974$

### Corrigé D7-65

1)  $X$  follows a  $B(6, 0.8)$

2)  $E(X) = n \times p = 6 \times 0.8 = 4.8$

3)  $P(X = 6) = \binom{6}{6} 0.8^6 0.2^0 = 0.8^6 = 0.262144$



4)  $P(X \geq 5) = P(X = 5) + P(X = 6) = 0.393216 + 0.262144 \approx \mathbf{0.655}$

5) Because of independence, what happens on Monday doesn't matter!  
 Let  $Y$  be the number of crosswords Jack completes from Tuesday to Saturday.  
 $Y$  follows a binomial distribution  $B(5, 0.8)$ .  
 We are looking for

$$P(Y = 4) = \binom{5}{4} 0.8^4 0.2^1 \approx \mathbf{0.410}$$

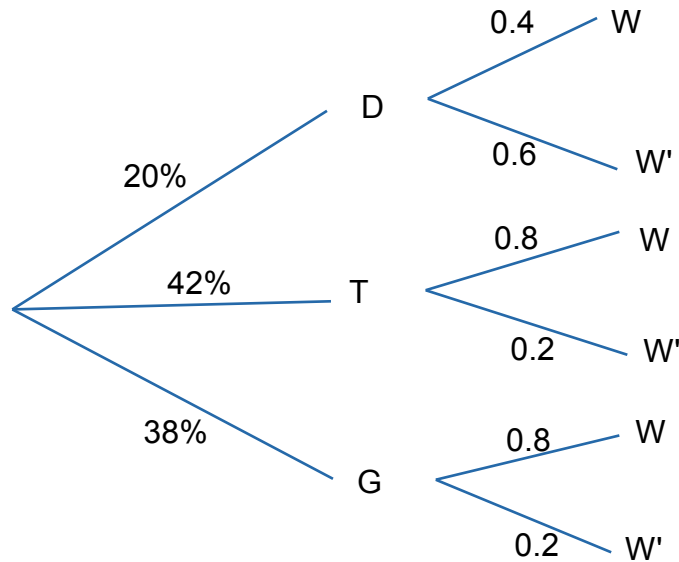
6) First we need  $P(X \leq 4) = 1 - P(X > 4) = 1 - P(X \geq 5) \approx 1 - 0.655 \approx \mathbf{0.345}$ .  
 Let  $Z$  be the number of weeks, in the period of four weeks, for which Jack completes 4 or fewer crosswords.  $Z$  follows a  $B(4, 0.345)$ .  
 We are asked to find

$$P(Z = 1) = \binom{4}{1} 0.345^1 0.655^3 \approx \mathbf{0.388}$$

**Corrigé D7-71**

ANSWERS

1)



2) T': "not having to deal with trolls": "having to deal with dragons or goblins".

3)  $P(G \cap W) = \frac{38}{100} \times 0.8 = 0.304$

4)  $P(W) = P(G \cap W) + P(D \cap W) + P(T \cap W) = 0.304 + 0.08 + 0.336 = 0.72$

5)  $\frac{P(T \cap W)}{P(W)} = \frac{0.336}{0.72} = 0.47(2dp)$

6) The probability that she gets a head is 0.85.  $P(W) = 0.72$

$P(H) \times P(W) = 0.612$

The probability that they get married is 0.612.

**Corrigé D7-72**

ANSWERS

1. (i)  $A = \{5; 7; 9; 11\}$  and  $P(A) = 4/12 = 1/3$ .  
 $B = \{1; 4; 9\}$  and  $P(B) = 3/12 = 1/4$ .

(ii)  $A \cap B = \{9\}$  and  $P(A \cap B) = 1/12$ .

$P(A)P(B) = (1/3)(1/4) = 1/12 = P(A \cap B)$ , and A and B are independent.

2. (i) The probability is:  $(1/12)(1/12) = 1/144$ .

(ii) We have to list all the possible ways of scoring 20:

$20 = 8+12 = 9+11 = 10+10 = 11+9 = 12+8$ .

Hence, the probability is  $5/144$ .

(ii) The probability is:  $1 - (11/12)^2 = 0.16$  to 2 d.p.

3. The corresponding probability is  $1 - (11/12)^{10} = 0.58$  to 2 d.p.

4. We must solve the inequality:  $1 - (11/12)^n > 0.99$ , with  $n$  positive integer.

We can use our calculator, or the logarithmic function: Jack should spin the wheel at least 53 times!

### Corrigé D7-73

#### ANSWERS

1) A. There are 40 sweets altogether, and  $6 + 8 + 2 = 16$  of them taste weird.

So  $P(\text{weird}) = 16/40 = 2/5$

B. There are  $5 + 2 = 7$  sweets that are blue, so 33 aren't blue.

$P(\text{not being blue}) = 33/40$

C. There are  $8 + 11 + 5 + 8 = 32$  sweets that are tasting good or being green

So  $P(\text{tasting normal or being green}) = 32/40$

D. Of the 14 red sweets, 6 taste weird,

So  $P(\text{tasting weird given that it's red}) = 6/14 = 3/7$

2)  $P(\text{Weird then Weird then Ordinary}) = 16/40 \times 15/39 \times 24/38 = 5760/59280 = 24/247$

But there are 3 (WWO, WOW, OWW) ways to select two weird beans and one ordinary bean, so

$P(\text{2 weird and 1 ordinary}) = 3 \times 24 / 247 = 72/247$ .

(one could draw a tree diagram)

### Corrigé D7-81

1) a) Number of possible outcomes :  $36 + 2 = 38$

Number of outcomes that satisfy our constraints : 1

$P(\{4\}) = \frac{1}{38} \approx 0.0263$  to 4d.p.

b) The player bets £1.

If "4" occurs, the player gets £1+£35.  $-1 + 1 + 35 = +35$ .

Else,  $-1 + 0 = -1$ .

The possible values for  $X$  are (+35) and (-1).

c)

amount ( $X = ?$ )	-1	+35
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probability $P(X = ?)$	$\frac{37}{38}$	$\frac{1}{38}$
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d)  $E(X) = \sum P(X = x_i) \times x_i = -1 \times \frac{37}{38} + 35 \times \frac{1}{38} = \frac{-2}{38} = \frac{-1}{19} \approx -0.0526$  to 4d.p.

This expected value is slightly negative, the casino keeps a small percentage of the bets (called "House Edge") while the rest of the money is given back to the players to pay their wins.

2) a) Number of possible outcomes :  $36 + 2 = 38$

Number of outcomes that satisfy our constraints (number of odd numbers) : 18

$P(\text{"odd"}) = \frac{18}{38} = \frac{9}{19} \approx 0.4737$  to 4d.p.

b) The player bets \$1.

If "odd" occurs, the player gets £1+£1.  $-1 + 1 + 1 = +1$ .

Else,  $-1 + 0 = -1$ .

The possible values for  $Y$  are (+1) and (-1).

c)

amount ( $Y = ?$ )	-1	+1
probability $P(Y = ?)$	$\frac{20}{38} = \frac{10}{19}$	$\frac{18}{38} = \frac{9}{19}$

d)  $E(Y) = \sum P(Y = y_i) \times y_i = -1 \times \frac{10}{19} + 1 \times \frac{9}{19} = \frac{-1}{19} \approx -0.0526$  to 4d.p.

This expected value is slightly negative (and the same as previously), the casino keeps a small percentage of the bets (called "House Edge") while the rest of the money is given back to the players to pay their wins.