Calcul formel

Calcul numérique ou formel

Etude d’une suite constante

On définit la suite $(u_n)_{n\in\mathbb{N}}$ par

$$\begin{cases} u_0 &= 0.2 \\ u_{n+1} &= 11\,u_n - 2 \quad \forall\, n \in \mathbb{N} \end{cases}.$$

Etudier la convergence de (u_n).

Mêmes questions avec les suites $(v_n)_{n\in\mathbb{N}}$ et $(w_n)_{n\in\mathbb{N}}$ définies respectivement par

$$\begin{cases} v_0 &= 0.3 \\ v_{n+1} &= 17\,v_n - 4.8 \quad \forall\, n \in \mathbb{N} \end{cases}$$

et

$$\begin{cases} w_0 &= 0.5 \\ w_{n+1} &= 13\,w_n - 6 \quad \forall\, n \in \mathbb{N} \end{cases}$$

Calculons les 25 premiers termes de la suite (u_n).

```
In[1]:= NestList[11 # - 2 &, 0.2, 25]
```

`Out[1]=`

$$\{0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.2, 0.200005, 0.200051, 0.200557, 0.206132, 0.267457, 0.942028, 8.36231, 89.9854, 987.84, 10864.2, 119505., 1.31455\times10^{6}, 1.446\times10^{7}, 1.5906\times10^{8}, 1.74966\times10^{9}\}$$

En prenant $\frac{2}{10}$ au lieu de 0,2, on obtient

```
In[2]:= NestList[11 # - 2 &, 2 / 10, 25]
```

`Out[2]=`

$$\left\{\frac{1}{5}, \frac{1}{5}\right\}$$

et le point fixe de cette suite est bien celui qui est attendu :

```
In[3]:= FixedPoint[11 # - 2 &, 2 / 10]
```

`Out[3]=`

$$\frac{1}{5}$$

Pour la seconde suiet (v_n), les 25 premiers termes sont

```
In[4]:= NestList[17 # - 4.8 &, 0.3, 25]
```

`Out[4]=`

$$\{0.3, 0.3, 0.3, 0.3, 0.3, 0.3, 0.3, 0.3, 0.3, 0.299999, 0.29998, 0.299664, 0.294293, 0.202974, -1.34944, -27.7405, -476.389, -8103.42, -137763., -2.34197\times10^{6}, -3.98136\times10^{7}, -6.7683\times10^{8}, -1.15061\times10^{10}, -1.95604\times10^{11}, -3.32527\times10^{12}, -5.65296\times10^{13}\}$$

ou, en prenant $\frac{3}{10}$ au lieu de 0,3 :

```
In[5]:= NestList[17 # - 48 / 10 &, 3 / 10, 25]
```

Out[5]=
$$\left\{\frac{3}{10},\frac{3}{10}\right\}$$

Enfin, pour la suite (w_n) :

```
NestList[13 # - 6 &, 0.5, 25]
```

On vas étudier les chiffres qui constituent le développement décimal de 0,2

```
In[6]:= RealDigits[0.2]

Out[6]= {{2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0}, 0}
```

ce qui, en base 2 donne

```
In[7]:= dig = RealDigits[0.2, 2]

Out[7]= {{1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1,
           1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 0}, −2}
```

Le nombre dont le développement est donné ci-dessus n'est pas 0,2 mais

```
In[8]:= res = FromDigits[dig, 2]
```

Out[8]=
$$\frac{3\,602\,879\,701\,896\,397}{18\,014\,398\,509\,481\,984}$$

dont une valeur approchée est

```
In[9]:= N[res]

Out[9]= 0.2
```

mais qui donne, en augmentant la précision

```
In[10]:= N[res, 25]

Out[10]= 0.2000000000000000111022302

In[11]:= Remove[dig, res]
```

Polynôme de Rump

Le polynôme de Rump P (ainsi dénommé mais ce n'est pas un polynôme !) est défini pour tout $(x,\ y) \in \mathbb{R} \times \mathbb{R}^*$ par

In[12]:=
$$\texttt{p[x\_, y\_] :=}\ 333.75\, y^6 + x^2 * \left(11\, x^2 * y^2 - y^6 - 121\, y^4 - 2\right) + 5.5\, y^8 + \frac{x}{2\, y};$$

```
p[x, y]
```

Out[13]=
$$x^2\left(11\, x^2\, y^2 - y^6 - 121\, y^4 - 2\right) + \frac{x}{2\, y} + 5.5\, y^8 + 333.75\, y^6$$

Déterminer l'image de $M\ (77\,617,\ 33\,096)$ par P.

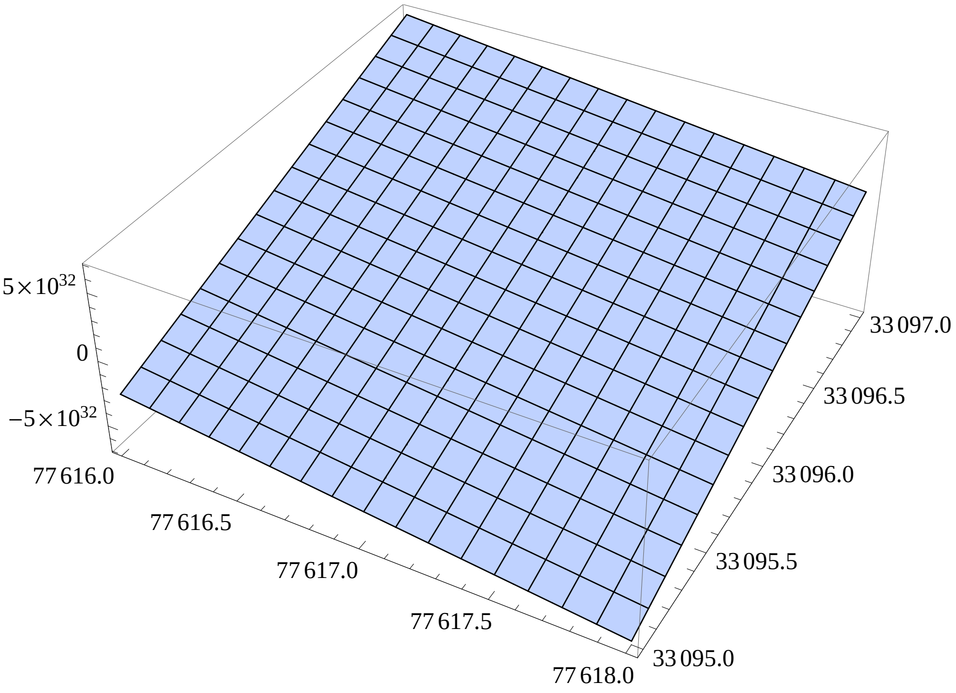
In[14]:=
```
{a, b} = {77 617, 33 096};
p[a, b]
```

Out[15]= 1.18059×10^{21}

Etudions la surface obtenue au voisinage de *M*

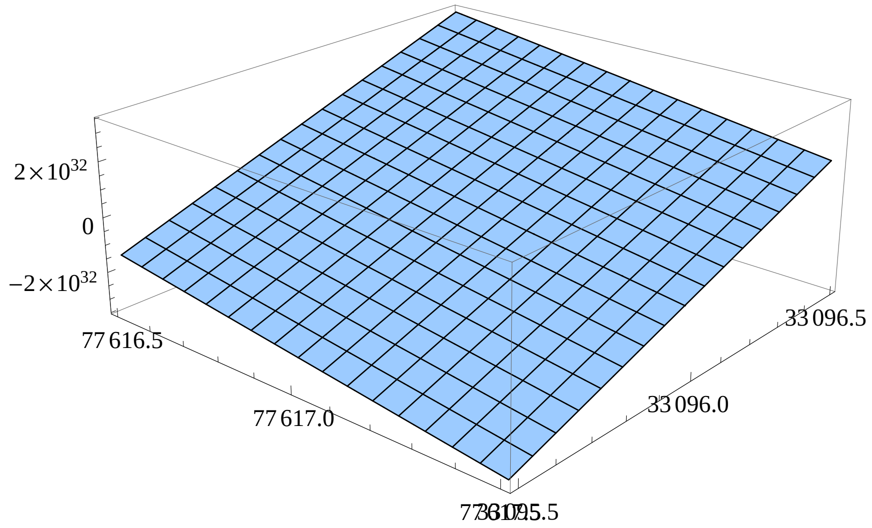
In[16]:=
```
err = 1;
Plot3D[p[x, y], {x, a - err, a + err}, {y, b - err, b + err}]
```

Out[17]=



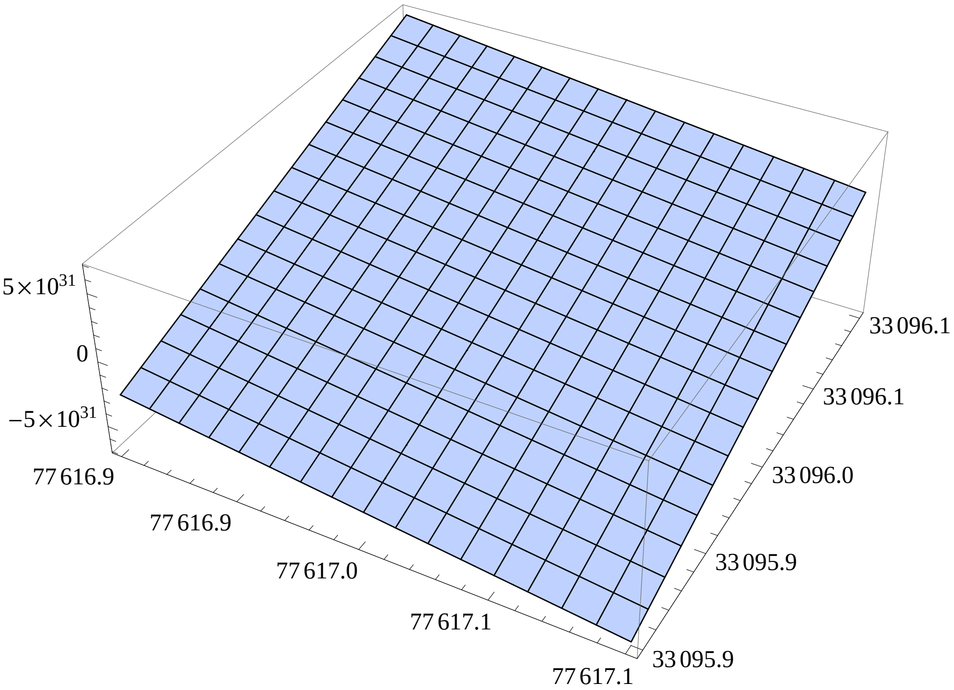
In[20]:=
```
err = 0.5;
Plot3D[p[x, y], {x, a - err, a + err}, {y, b - err, b + err}]
```

Out[21]=



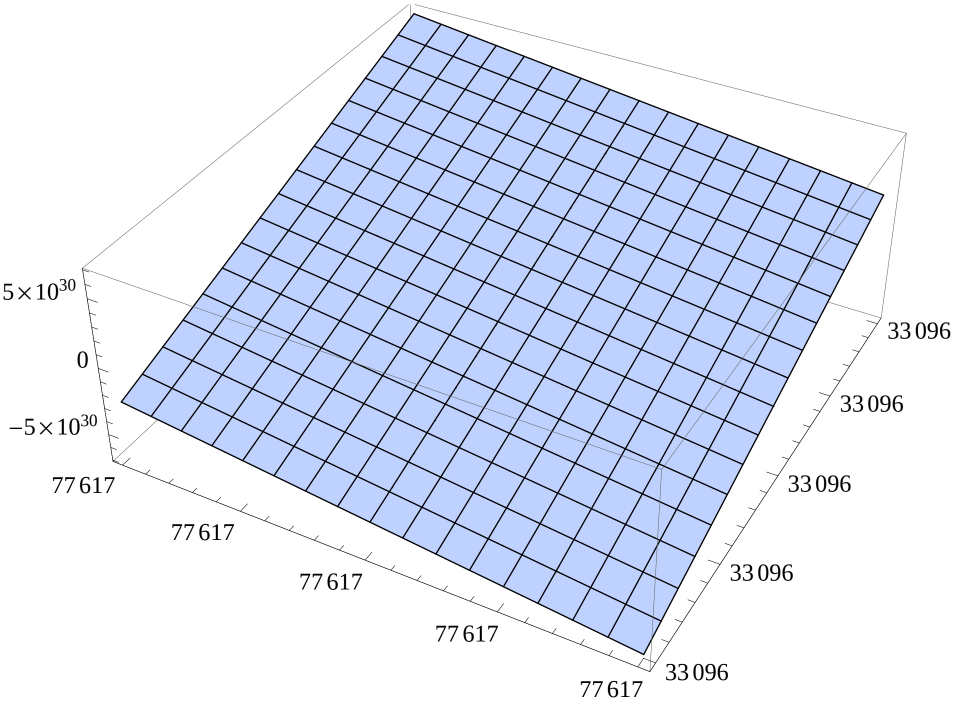
```
In[22]:= err = 0.1;
         Plot3D[p[x, y], {x, a - err, a + err}, {y, b - err, b + err}]
```

Out[23]=



```
In[30]:= err = 0.01;
         Plot3D[p[x, y], {x, a - err, a + err}, {y, b - err, b + err}]
```

Out[31]=



Voilà qui est pour le moins curieux !
Si l’on choisit de travailler en virgule flottante avec tous les coefficients intervenant dans *P* on obtient

```
In[32]:=  r1[x_, y_] = N[p[x, y]]
```

Out[32]= $$x^2\left(11.\,x^2\,y^2 - 1.\,y^6 - 121.\,y^4 - 2.\right) + \frac{0.5\,x}{y} + 5.5\,y^8 + 333.75\,y^6$$

```
In[33]:=  r1[a, b]
```

Out[33]= $$-1.39061\times 10^{21}$$

Que choisir ?
Et si l’on choisit de travailler en valeur exacte :

```
In[34]:=  r[x_, y_] = Rationalize[p[x, y]]
```

Out[34]= $$x^2\left(11\,x^2\,y^2 - y^6 - 121\,y^4 - 2\right) + \frac{x}{2\,y} + \frac{11\,y^8}{2} + \frac{1335\,y^6}{4}$$

```
In[35]:=  r[a, b]
```

Out[35]= $$-\frac{54\,767}{66\,192}$$

Résultat qui semble plus proche de ce qui peut être attendu au vu des premières représentations graphiques obtenues.

```
In[37]:=  Remove[a, b, p, err, r, r1]
```

On définit la fonction de deux variables *x*, *y* en indiquant les différents coefficients choisis comme paramètres et on va recueillir les différentes valeurs obtenues suivant que l’on choisit un coefficient en virgule flottante ou en valeur exacte.

```
In[38]:=  p[c1_, c2_, c3_, c4_, c5_, c6_, x_, y_] :=
```

$$\mathtt{c1 * y^6 + x^2 * \left(c2 * x^2 * y^2 - y^6 - c3 * y^4 - c4\right) + c5 * y^8 + \frac{x}{c6 * y};}$$

```
In[39]:=  coef = {33 375 / 100, 11, 121, 2, 55 / 10, 2, 77 617, 33 096};
```

En valeur exacte, on obtient bien

```
In[40]:=  p @@ coef
```

Out[40]= $$-\frac{54\,767}{66\,192}$$

Les différents coefficients sont

```
In[41]:=  coef2 = {#, N@#} & /@ coef
```

Out[41]= $$\begin{pmatrix} \frac{1335}{4} & 333.75 \\ 11 & 11. \\ 121 & 121. \\ 2 & 2. \\ \frac{11}{2} & 5.5 \\ 2 & 2. \\ 77\,617 & 77\,617. \\ 33\,096 & 33\,096. \end{pmatrix}$$

ce qui donne les 256 possibilités suivantes

```
In[42]:= prec = Tuples[coef2]
```

$$\left(\begin{array}{cccccccc}
\frac{1335}{4} & 11 & 121 & 2 & \frac{11}{2} & 2 & 77\,617 & 33\,096 \\
\frac{1335}{4} & 11 & 121 & 2 & \frac{11}{2} & 2 & 77\,617 & 33\,096. \\
\frac{1335}{4} & 11 & 121 & 2 & \frac{11}{2} & 2 & 77\,617. & 33\,096 \\
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\end{array}\right.$$

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1335							

Out[42]=

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1335/4  11.  121.  2.  5.5   2.  77 617.  33 096.
333.75  11   121   2   11/2  2   77 617   33 096
333.75  11   121   2   11/2  2   77 617   33 096.
333.75  11   121   2   11/2  2   77 617.  33 096
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333.75  11   121   2   5.5   2.  77 617   33 096
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\left|\begin{array}{cccccccc}
333.75 & 11 & 121 & 2 & 5.5 & 2. & 77\,617. & 33\,096 \\
333.75 & 11 & 121 & 2 & 5.5 & 2. & 77\,617. & 33\,096. \\
333.75 & 11 & 121 & 2. & \frac{11}{2} & 2 & 77\,617 & 33\,096 \\
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Et les différentes valeurs obtenues en calculant avec l’une ou l’autre de ces listes de coefficients est

```
In[43]:= DeleteDuplicates[p @@@ prec]
```

Out[43]= $\left\{-\frac{54\,767}{66\,192}, -1.18059\times 10^{21}, -2.36118\times 10^{21}, 0., 1.18059\times 10^{21}\right\}$

```
In[44]:= Remove[p, coef, coef2, prec]
```

Calcul des termes d’une suite définie par une relation de récurrence d’ordre 2

Déterminer les 100 premiers termes de la suite (u_n) définie par la relation de récurrence

$$
\begin{cases}
u_0 & = & 2 \\
u_1 & = & -4 \\
u_{n+1} & = & 111 - \frac{1130}{u_n} + \frac{3000}{u_n \times u_{n-1}}
\end{cases}
$$

Calculons ces termes en choisissant les valeurs exactes.

```
In[45]:= res = RecurrenceTable[{u[n] == 111 - 1130 / u[n - 1] + 3000 / (u[n - 1] * u[n - 2]),
            u[0] == 2, u[1] == -4}, u, {n, 100}]
```

`Out[45]=`

$$\Big\{2,\ -4,\ \frac{37}{2},\ \frac{347}{37},\ \frac{2707}{347},\ \frac{19\,367}{2707},\ \frac{131\,827}{19\,367},\ \frac{869\,087}{131\,827},\ \frac{5\,605\,147}{869\,087},\ \frac{35\,584\,007}{5\,605\,147},\ \frac{223\,269\,667}{35\,584\,007},$$

$$\frac{1\,388\,446\,127}{223\,269\,667},\ \frac{8\,574\,817\,387}{1\,388\,446\,127},\ \frac{52\,669\,607\,447}{8\,574\,817\,387},\ \frac{322\,121\,160\,307}{52\,669\,607\,447},\ \frac{1\,963\,244\,539\,967}{322\,121\,160\,307},$$

$$\frac{11\,932\,055\,130\,427}{1\,963\,244\,539\,967},\ \frac{72\,355\,270\,235\,687}{11\,932\,055\,130\,427},\ \frac{437\,946\,318\,679\,747}{72\,355\,270\,235\,687},\ \frac{2\,646\,751\,398\,406\,607}{437\,946\,318\,679\,747},$$

$$\frac{15\,975\,875\,822\,080\,267}{2\,646\,751\,398\,406\,607},\ \frac{96\,332\,092\,090\,684\,727}{15\,975\,875\,822\,080\,267},\ \frac{580\,376\,738\,335\,123\,987}{96\,332\,092\,090\,684\,727},$$

$$\frac{3\,494\,181\,358\,965\,822\,047}{580\,376\,738\,335\,123\,987},\ \frac{21\,024\,692\,798\,570\,322\,907}{3\,494\,181\,358\,965\,822\,047},\ \frac{126\,446\,180\,015\,298\,890\,567}{21\,024\,692\,798\,570\,322\,907},$$

$$\frac{760\,167\,196\,211\,178\,109\,027}{126\,446\,180\,015\,298\,890\,567},\ \frac{4\,568\,453\,757\,863\,992\,482\,287}{760\,167\,196\,211\,178\,109\,027},\ \frac{27\,447\,975\,450\,168\,574\,034\,347}{4\,568\,453\,757\,863\,992\,482\,287},$$

$$\frac{164\,874\,117\,215\,934\,539\,909\,207}{27\,447\,975\,450\,168\,574\,034\,347},\ \frac{990\,176\,025\,870\,222\,717\,970\,867}{164\,874\,117\,215\,934\,539\,909\,207},\ \frac{5\,945\,712\,768\,094\,413\,700\,403\,327}{990\,176\,025\,870\,222\,717\,970\,867},$$

$$\frac{35\,697\,559\,672\,931\,869\,165\,310\,587}{5\,945\,712\,768\,094\,413\,700\,403\,327},\ \frac{214\,301\,773\,359\,418\,149\,806\,316\,647}{35\,697\,559\,672\,931\,869\,165\,310\,587},$$

$$\frac{1\,286\,392\,716\,765\,643\,572\,910\,165\,507}{214\,301\,773\,359\,418\,149\,806\,316\,647},\ \frac{7\,721\,266\,683\,639\,534\,807\,822\,321\,167}{1\,286\,392\,716\,765\,643\,572\,910\,165\,507},$$

$$\frac{46\,342\,152\,017\,065\,575\,698\,740\,567\,627}{7\,721\,266\,683\,639\,534\,807\,822\,321\,167},\ \frac{278\,125\,671\,678\,535\,288\,451\,476\,608\,887}{46\,342\,152\,017\,065\,575\,698\,740\,567\,627},$$

$$\frac{1\,669\,117\,827\,951\,920\,902\,004\,025\,668\,947}{278\,125\,671\,678\,535\,288\,451\,476\,608\,887},\ \frac{10\,016\,525\,957\,115\,071\,268\,499\,984\,091\,807}{1\,669\,117\,827\,951\,920\,902\,004\,025\,668\,947},$$

$$\frac{60\,108\,250\,689\,708\,156\,893\,379\,054\,941\,467}{10\,016\,525\,957\,115\,071\,268\,499\,984\,091\,807},\ \frac{360\,694\,978\,873\,337\,587\,772\,170\,081\,601\,927}{60\,108\,250\,689\,708\,156\,893\,379\,054\,941\,467},$$

$$\frac{2\,164\,397\,246\,915\,468\,758\,692\,499\,249\,377\,187}{360\,694\,978\,873\,337\,587\,772\,170\,081\,601\,927},\ \frac{12\,987\,520\,349\,870\,028\,712\,452\,389\,295\,091\,247}{2\,164\,397\,246\,915\,468\,758\,692\,499\,249\,377\,187},$$

$$\frac{77\,930\,806\,441\,106\,253\,076\,201\,304\,764\,688\,107}{12\,987\,520\,349\,870\,028\,712\,452\,389\,295\,091\,247},\ \frac{467\,613\,260\,356\,067\,922\,464\,642\,673\,558\,831\,767}{77\,930\,806\,441\,106\,253\,076\,201\,304\,764\,688\,107},$$

$$\frac{2\,805\,821\,670\,683\,559\,554\,825\,030\,266\,206\,506\,227}{467\,613\,260\,356\,067\,922\,464\,642\,673\,558\,831\,767},\ \frac{16\,835\,640\,566\,837\,117\,429\,136\,052\,721\,506\,615\,487}{2\,805\,821\,670\,683\,559\,554\,825\,030\,266\,206\,506\,227},$$

$$\frac{101\,017\,396\,114\,701\,505\,075\,745\,671\,950\,377\,583\,547}{16\,835\,640\,566\,837\,117\,429\,136\,052\,721\,506\,615\,487},$$

$$\frac{606\,122\,140\,256\,603\,032\,959\,120\,809\,808\,954\,954\,407}{101\,017\,396\,114\,701\,505\,075\,745\,671\,950\,377\,583\,547},$$

$$\frac{3\,636\,821\,659\,381\,588\,210\,277\,958\,749\,387\,176\,992\,067}{606\,122\,140\,256\,603\,032\,959\,120\,809\,808\,954\,954\,407},$$

$$\frac{21\,821\,374\,045\,499\,379\,324\,283\,921\,948\,990\,298\,280\,527}{3\,636\,821\,659\,381\,588\,210\,277\,958\,749\,387\,176\,992\,067},$$

$$\frac{130\,930\,464\,719\,045\,526\,258\,784\,378\,957\,277\,971\,323\,787}{21\,821\,374\,045\,499\,379\,324\,283\,921\,948\,990\,298\,280\,527},$$

$$\frac{785\,593\,890\,544\,519\,409\,118\,110\,510\,060\,348\,736\,145\,847}{130\,930\,464\,719\,045\,526\,258\,784\,378\,957\,277\,971\,323\,787},$$

$$\frac{4\,713\,618\,854\,418\,347\,712\,535\,684\,241\,945\,496\,957\,890\,707}{785\,593\,890\,544\,519\,409\,118\,110\,510\,060\,348\,736\,145\,847},$$

$$\frac{28\,281\,990\,682\,266\,242\,564\,349\,211\,359\,590\,004\,452\,422\,367}{4\,713\,618\,854\,418\,347\,712\,535\,684\,241\,945\,496\,957\,890\,707},$$

$$\frac{169\,693\,331\,872\,378\,236\,831\,770\,797\,697\,125\,140\,239\,924\,827}{28\,281\,990\,682\,266\,242\,564\,349\,211\,359\,590\,004\,452\,422\,367},$$

$$\frac{1\,018\,166\,930\,128\,173\,328\,219\,002\,433\,880\,676\,409\,066\,502\,087}{169\,693\,331\,872\,378\,236\,831\,770\,797\,697\,125\,140\,239\,924\,827},$$

$$\frac{6\,109\,036\,275\,238\,559\,505\,455\,902\,841\,773\,686\,292\,533\,778\,147}{1\,018\,166\,930\,128\,173\,328\,219\,002\,433\,880\,676\,409\,066\,502\,087},$$

$$\frac{36\,654\,391\,123\,778\,954\,713\,444\,858\,243\,090\,256\,945\,876\,497\,007}{6\,109\,036\,275\,238\,559\,505\,455\,902\,841\,773\,686\,292\,533\,778\,147},$$

$$\frac{219\,927\,214\,104\,411\,716\,684\,216\,355\,420\,782\,237\,628\,628\,122\,667}{36\,654\,391\,123\,778\,954\,713\,444\,858\,243\,090\,256\,945\,876\,497\,007},$$

$$\frac{1\,319\,567\,621\,435\,160\,242\,123\,034\,162\,335\,896\,905\,538\,614\,439\,127}{219\,927\,214\,104\,411\,716\,684\,216\,355\,420\,782\,237\,628\,628\,122\,667},$$

$$\frac{7\,917\,427\,412\,654\,411\,162\,826\,885\,123\,071\,398\,832\,065\,915\,150\,387}{1\,319\,567\,621\,435\,160\,242\,123\,034\,162\,335\,896\,905\,538\,614\,439\,127},$$

$$\frac{47\,504\,672\,896\,143\,715\,527\,404\,711\,483\,708\,479\,986\,566\,633\,480\,447}{7\,917\,427\,412\,654\,411\,162\,826\,885\,123\,071\,398\,832\,065\,915\,150\,387},$$

$$\frac{285\,028\,579\,477\,948\,535\,916\,645\,272\,628\,651\,314\,890\,255\,513\,773\,307}{47\,504\,672\,896\,143\,715\,527\,404\,711\,483\,708\,479\,986\,566\,633\,480\,447},$$

$$\frac{1\,710\,174\,187\,373\,122\,429\,260\,956\,654\,403\,910\,064\,195\,811\,647\,092\,967}{285\,028\,579\,477\,948\,535\,916\,645\,272\,628\,651\,314\,890\,255\,513\,773\,307},$$

$$\frac{10\,261\,058\,676\,765\,890\,644\,371\,165\,019\,583\,471\,259\,446\,262\,704\,823\,427}{1\,710\,174\,187\,373\,122\,429\,260\,956\,654\,403\,910\,064\,195\,811\,647\,092\,967},$$

$$\frac{61\,566\,419\,823\,231\,124\,210\,254\,115\,583\,300\,881\,928\,034\,540\,340\,268\,687}{10\,261\,058\,676\,765\,890\,644\,371\,165\,019\,583\,471\,259\,446\,262\,704\,823\,427},$$

$$\frac{369\,398\,857\,752\,565\,646\,981\,660\,320\,828\,805\,563\,424\,992\,062\,598\,252\,747}{61\,566\,419\,823\,231\,124\,210\,254\,115\,583\,300\,881\,928\,034\,540\,340\,268\,687},$$

$$\frac{2\,216\,394\,840\,581\,288\,390\,490\,640\,061\,617\,834\,739\,833\,876\,478\,372\,719\,607}{369\,398\,857\,752\,565\,646\,981\,660\,320\,828\,805\,563\,424\,992\,062\,598\,252\,747},$$

$$\frac{13\,298\,377\,513\,817\,202\,885\,947\,231\,052\,932\,015\,235\,422\,879\,384\,152\,333\,267}{2\,216\,394\,840\,581\,288\,390\,490\,640\,061\,617\,834\,739\,833\,876\,478\,372\,719\,607},$$

$$\frac{79\,790\,307\,434\,550\,580\,030\,700\,339\,733\,717\,125\,394\,635\,378\,874\,494\,077\,727}{13\,298\,377\,513\,817\,202\,885\,947\,231\,052\,932\,015\,235\,422\,879\,384\,152\,333\,267},$$

$$\frac{478\,742\,056\,365\,540\,293\,759\,286\,805\,482\,927\,922\,278\,302\,786\,094\,864\,856\,987}{79\,790\,307\,434\,550\,580\,030\,700\,339\,733\,717\,125\,394\,635\,378\,874\,494\,077\,727},$$

$$\frac{2\,872\,453\,396\,984\,425\,830\,431\,144\,668\,300\,693\,383\,222\,269\,280\,808\,691\,095\,047}{478\,742\,056\,365\,540\,293\,759\,286\,805\,482\,927\,922\,278\,302\,786\,094\,864\,856\,987},$$

$$\frac{17\,234\,725\,675\,862\,475\,321\,963\,987\,186\,819\,789\,547\,095\,878\,506\,049\,656\,335\,907}{2\,872\,453\,396\,984\,425\,830\,431\,144\,668\,300\,693\,383\,222\,269\,280\,808\,691\,095\,047},$$

$$\frac{103\,408\,380\,524\,954\,453\,628\,669\,519\,005\,996\,883\,521\,386\,585\,142\,285\,486\,843\,567}{17\,234\,725\,675\,862\,475\,321\,963\,987\,186\,819\,789\,547\,095\,878\,506\,049\,656\,335\,907},$$

$$\frac{620\,450\,415\,498\,624\,730\,256\,445\,093\,461\,372\,032\,322\,376\,081\,383\,650\,665\,202\,027}{103\,408\,380\,524\,954\,453\,628\,669\,519\,005\,996\,883\,521\,386\,585\,142\,285\,486\,843\,567},$$

$$\frac{3\,722\,703\,154\,736\,238\,423\,960\,810\,457\,895\,185\,849\,904\,539\,340\,951\,592\,711\,915\,287}{620\,450\,415\,498\,624\,730\,256\,445\,093\,461\,372\,032\,322\,376\,081\,383\,650\,665\,202\,027},$$

$$\frac{22\,336\,222\,237\,139\,880\,755\,875\,562\,233\,005\,883\,379\,278\,650\,308\,957\,999\,875\,007\,347}{3\,722\,703\,154\,736\,238\,423\,960\,810\,457\,895\,185\,849\,904\,539\,340\,951\,592\,711\,915\,287},$$

$$\frac{134\,017\,349\,966\,451\,535\,595\,806\,870\,826\,209\,141\,674\,928\,973\,169\,990\,217\,267\,622\,207}{22\,336\,222\,237\,139\,880\,755\,875\,562\,233\,005\,883\,379\,278\,650\,308\,957\,999\,875\,007\,347},$$

$$\frac{804\,104\,182\,516\,770\,468\,877\,608\,712\,098\,124\,057\,045\,859\,195\,601\,152\,393\,693\,623\,867}{134\,017\,349\,966\,451\,535\,595\,806\,870\,826\,209\,141\,674\,928\,973\,169\,990\,217\,267\,622\,207},$$

$$\frac{4\,824\,625\,508\,690\,929\,089\,779\,489\,708\,293\,090\,377\,256\,581\,956\,512\,969\,812\,601\,196\,327}{804\,104\,182\,516\,770\,468\,877\,608\,712\,098\,124\,057\,045\,859\,195\,601\,152\,393\,693\,623\,867},$$

$$\frac{28\,947\,755\,120\,097\,105\,921\,246\,125\,428\,280\,272\,438\,446\,625\,653\,608\,096\,127\,804\,443\,587}{4\,824\,625\,508\,690\,929\,089\,779\,489\,708\,293\,090\,377\,256\,581\,956\,512\,969\,812\,601\,196\,327},$$

$$\frac{173\,686\,541\,060\,340\,292\,440\,322\,688\,462\,290\,285\,505\,215\,423\,494\,299\,963\,027\,812\,989\,647}{28\,947\,755\,120\,097\,105\,921\,246\,125\,428\,280\,272\,438\,446\,625\,653\,608\,096\,127\,804\,443\,587},$$

$$\frac{1\,042\,119\,298\,060\,830\,039\,206\,165\,810\,236\,784\,967\,403\,970\,888\,829\,056\,709\,471\,809\,578\,507}{173\,686\,541\,060\,340\,292\,440\,322\,688\,462\,290\,285\,505\,215\,423\,494\,299\,963\,027\,812\,989\,647},$$

$$\frac{6\,252\,716\,046\,858\,921\,658\,058\,143\,258\,735\,926\,076\,287\,217\,072\,290\,624\,913\,355\,515\,674\,167}{1\,042\,119\,298\,060\,830\,039\,206\,165\,810\,236\,784\,967\,403\,970\,888\,829\,056\,709\,471\,809\,578\,507},$$

$$\frac{37\,516\,297\,573\,623\,237\,062\,454\,601\,538\,991\,637\,817\,040\,261\,130\,325\,172\,762\,756\,385\,060\,627}{6\,252\,716\,046\,858\,921\,658\,058\,143\,258\,735\,926\,076\,287\,217\,072\,290\,624\,913\,355\,515\,674\,167},$$

$$\frac{225\,097\,791\,904\,087\,957\,945\,256\,319\,166\,830\,233\,698\,826\,360\,264\,858\,152\,989\,654\,765\,441\,887}{37\,516\,297\,573\,623\,237\,062\,454\,601\,538\,991\,637\,817\,040\,261\,130\,325\,172\,762\,756\,385\,060\,627},$$

$$\frac{1\,350\,586\,783\,736\,270\,425\,524\,181\,464\,665\,383\,436\,175\,882\,129\,003\,684\,500\,003\,510\,868\,041\,947}{225\,097\,791\,904\,087\,957\,945\,256\,319\,166\,830\,233\,698\,826\,360\,264\,858\,152\,989\,654\,765\,441\,887},$$

$$\frac{8\,103\,520\,863\,976\,335\,942\,408\,306\,536\,314\,310\,786\,969\,912\,611\,094\,784\,910\,348\,976\,585\,204\,807}{1\,350\,586\,783\,736\,270\,425\,524\,181\,464\,665\,383\,436\,175\,882\,129\,003\,684\,500\,003\,510\,868\,041\,947},$$

$$\frac{48\,621\,125\,991\,651\,582\,600\,765\,927\,959\,495\,915\,571\,392\,574\,851\,932\,099\,013\,733\,416\,395\,994\,467}{8\,103\,520\,863\,976\,335\,942\,408\,306\,536\,314\,310\,786\,969\,912\,611\,094\,784\,910\,348\,976\,585\,204\,807},$$

$$\frac{291\,726\,759\,988\,877\,330\,336\,176\,011\,465\,025\,747\,676\,220\,945\,038\,409\,541\,840\,598\,282\,799\,794\,927}{48\,621\,125\,991\,651\,582\,600\,765\,927\,959\,495\,915\,571\,392\,574\,851\,932\,099\,013\,733\,416\,395\,994\,467},$$

$$\frac{1\,750\,360\,580\,128\,103\,155\,674\,958\,287\,330\,405\,757\,296\,653\,149\,864\,541\,989\,834\,578\,618\,917\,910\,187}{291\,726\,759\,988\,877\,330\,336\,176\,011\,465\,025\,747\,676\,220\,945\,038\,409\,541\,840\,598\,282\,799\,794\,927},$$

$$\frac{10\,502\,163\,581\,742\,814\,802\,339\,260\,816\,683\,690\,899\,976\,556\,297\,357\,675\,632\,962\,416\,324\,103\,164\,247}{1\,750\,360\,580\,128\,103\,155\,674\,958\,287\,330\,405\,757\,296\,653\,149\,864\,541\,989\,834\,578\,618\,917\,910\,187},$$

$$\frac{63\,012\,981\,995\,327\,868\,155\,483\,120\,363\,608\,427\,180\,842\,524\,774\,998\,172\,267\,549\,220\,997\,597\,501\,107}{10\,502\,163\,581\,742\,814\,802\,339\,260\,816\,683\,690\,899\,976\,556\,297\,357\,675\,632\,962\,416\,324\,103\,164\,247},$$

$$\frac{378\,077\,894\,496\,322\,105\,640\,136\,499\,499\,181\,971\,989\,971\,083\,604\,249\,625\,954\,168\,941\,250\,477\,584\,767}{63\,012\,981\,995\,327\,868\,155\,483\,120\,363\,608\,427\,180\,842\,524\,774\,998\,172\,267\,549\,220\,997\,597\,501\,107},$$

$$\frac{2\,268\,467\,379\,599\,707\,117\,377\,007\,883\,582\,748\,876\,464\,406\,176\,396\,800\,717\,469\,381\,723\,827\,328\,399\,227}{378\,077\,894\,496\,322\,105\,640\,136\,499\,499\,181\,971\,989\,971\,083\,604\,249\,625\,954\,168\,941\,250\,477\,584\,767},$$

$$\frac{13\,610\,804\,340\,707\,115\,121\,942\,991\,734\,434\,778\,481\,409\,335\,432\,237\,319\,113\,538\,130\,724\,586\,284\,848\,487}{2\,268\,467\,379\,599\,707\,117\,377\,007\,883\,582\,748\,876\,464\,406\,176\,396\,800\,717\,469\,381\,723\,827\,328\,399\,227},$$

$$\frac{81\,664\,826\,359\,787\,052\,820\,062\,672\,571\,300\,097\,001\,570\,504\,462\,706\,488\,724\,837\,986\,255\,629\,281\,356\,547}{13\,610\,804\,340\,707\,115\,121\,942\,991\,734\,434\,778\,481\,409\,335\,432\,237\,319\,113\,538\,130\,724\,586\,284\,848\,487},$$

$$\frac{489\,988\,959\,736\,444\,127\,362\,399\,646\,251\,257\,712\,574\,995\,486\,122\,651\,802\,567\,073\,927\,074\,333\,549\,467\,407}{81\,664\,826\,359\,787\,052\,820\,062\,672\,571\,300\,097\,001\,570\,504\,462\,706\,488\,724\,837\,986\,255\,629\,281\,356\,547}$$

$$,$$

$$2\,939\,933\,766\,307\,273\,816\,384\,515\,931\,624\,831\,928\,277\,835\,213\,467\,975\,166\,492\,673\,610\,148\,790\,603\,445\ddots$$
$$067/$$
$$489\,988\,959\,736\,444\,127\,362\,399\,646\,251\,257\,712\,574\,995\,486\,122\,651\,802\,567\,073\,927\,074\,333\,549\,467\ddots$$
$$407\}$$

ce qui donne, en valeur approchée

```
In[46]:= N@res

Out[46]= {2., −4., 18.5, 9.37838, 7.80115, 7.15441, 6.80678, 6.59263, 6.44947, 6.34845, 6.27444,
   6.2187, 6.17584, 6.14236, 6.11588, 6.09474, 6.07772, 6.06394, 6.05272, 6.04355, 6.03603,
   6.02985, 6.02475, 6.02054, 6.01706, 6.01417, 6.01178, 6.0098, 6.00815, 6.00679, 6.00565,
   6.0047, 6.00392, 6.00326, 6.00272, 6.00226, 6.00188, 6.00157, 6.00131, 6.00109, 6.00091,
   6.00076, 6.00063, 6.00053, 6.00044, 6.00036, 6.0003, 6.00025, 6.00021, 6.00018, 6.00015,
   6.00012, 6.0001, 6.00008, 6.00007, 6.00006, 6.00005, 6.00004, 6.00003, 6.00003, 6.00002,
   6.00002, 6.00002, 6.00001, 6.00001, 6.00001, 6.00001, 6.00001, 6.00001, 6., 6., 6., 6., 6.,
   6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6., 6.}
```

Qu’obtient-on si l’on remplace 111 par 111.0 ?

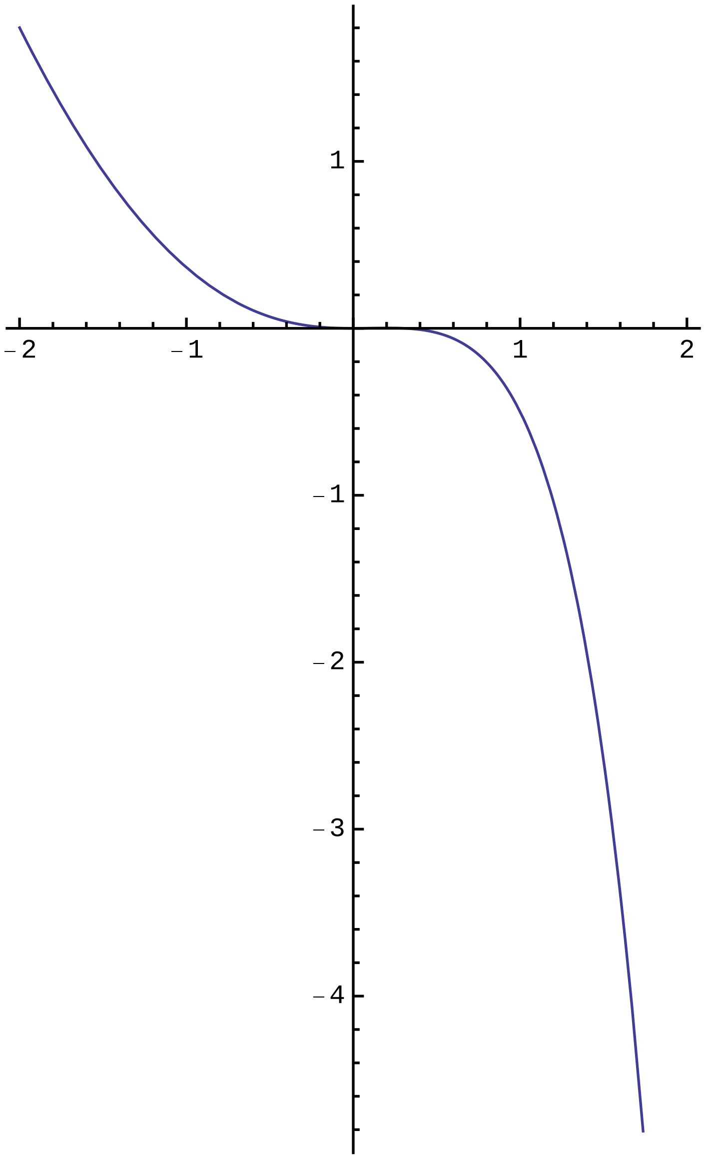
```
In[47]:= RecurrenceTable[{u[n] == 111. - 1130 / u[n - 1] + 3000 / (u[n - 1] * u[n - 2]),
    u[0] == 2, u[1] == -4}, u, {n, 100}]

Out[47]= {2., −4., 18.5, 9.37838, 7.80115, 7.15441, 6.80678, 6.59263, 6.44947, 6.34845, 6.27444,
   6.21868, 6.17563, 6.13898, 6.06081, 5.18585, −11.4519, 159.158, 102.254, 100.133, 100.008,
   100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100.,
   100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100.,
   100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100.,
   100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100.,
   100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100., 100.}
```

Analyse

```
In[50]:= f[x_] := x^2 * (1/2 - ⅇ^(x-1))
```

```
In[51]:= Plot[f@x, {x, -2, 2}, AspectRatio → Automatic]
```

Out[51]=



L’expression algébrique de la dérivée de f est donnée par

```
In[52]:= Factor[f'[x]]
```

Out[52]= $-\frac{x\,(e^x\,x + 2\,e^x - e)}{e}$

et les solutions de l’équation $f'(x) = 0$ sont

```
In[53]:= sol = Solve[f'[x] == 0, x]
```

```
Solve::ifun :
  Inverse functions are being used by Solve, so some solutions may not be found;
     use Reduce for complete solution information. ≫
```

Out[53]= $\{\{x \to 0\}, \{x \to W(e^3) - 2\}\}$

Que représente la dernière solution ?

```
In[54]:= InputForm@Last@sol
```

```
Out[54]//InputForm=
          {x -> -2 + ProductLog[E^3]}
```

```
In[55]:= ?ProductLog
```

ProductLog[z] gives the principal solution for w in $z = we^{w}$.

ProductLog[k, z] gives the k^{th} solution. ≫

L’extremum obtenu avec cette solution est

```
In[56]:= f[x] /. Last@sol
```

Out[56]= $$\left(\frac{1}{2} - e^{W(e^3)-3}\right)\left(W(e^3) - 2\right)^2$$

```
In[57]:= extreme = FullSimplify[f[x] /. Last@sol]
```

Out[57]= $$\frac{\left(W(e^3) - 2\right)^3}{2\,W(e^3)}$$

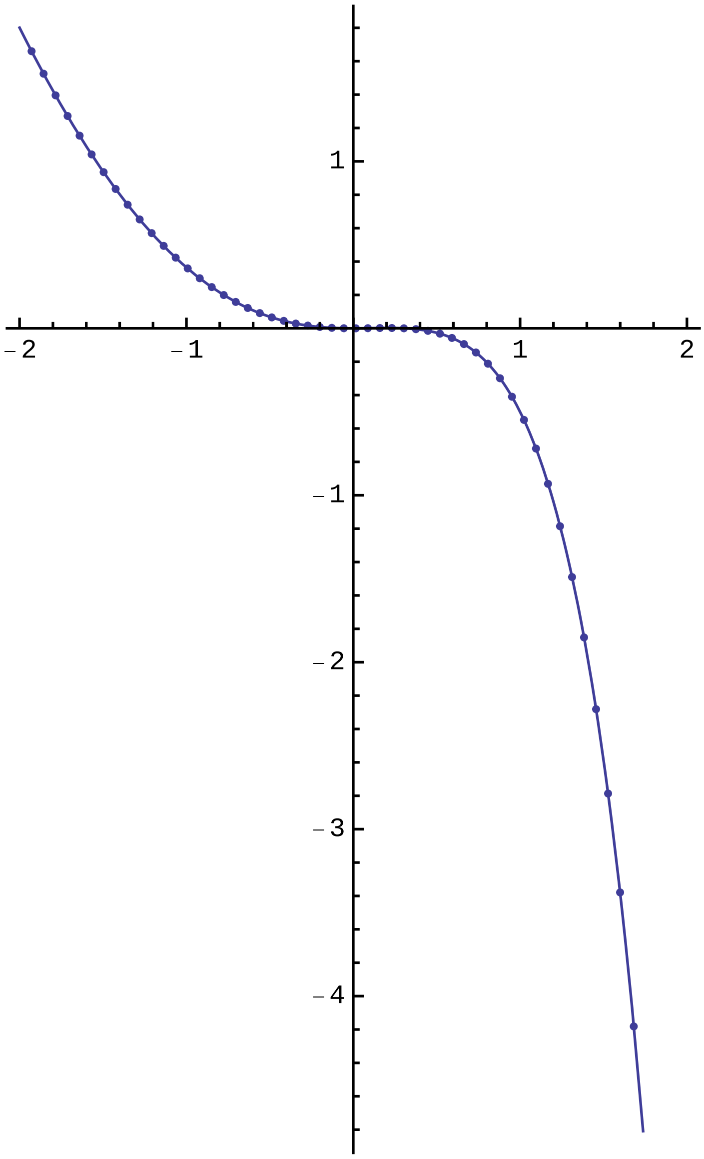
dont une valeur approchée est

```
In[58]:= N[extreme, 10]
```

Out[58]= 0.002036090359

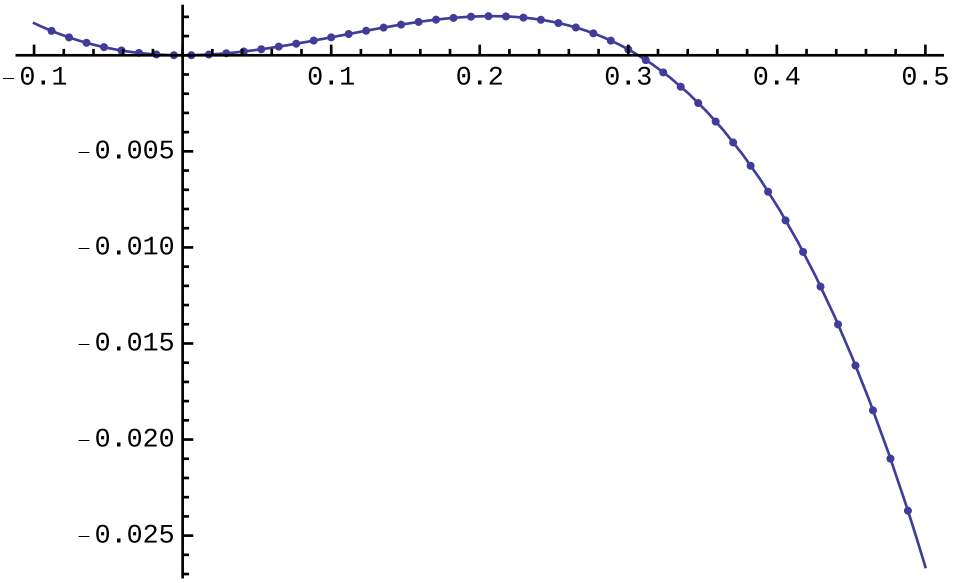
```
In[59]:=  Plot[f@x, {x, -2, 2}, AspectRatio → Automatic, Mesh → True]
```

Out[59]=



```
In[61]:= Plot[f@x, {x, -0.1, 0.5}, Mesh → True]
```

Out[61]=



Raisonnement, logique

Du bon usage des quantificateurs

```
In[62]:= ?Exists
```

Exists[*x*, *expr*] represents the statement
that there exists a value of *x* for which *expr* is True.
Exists[*x*, *cond*, *expr*] states that there exists an *x* satisfying
the condition *cond* for which *expr* is True.
Exists[{x_1, x_2, …}, *expr*] states that there exist values
for all the x_i for which *expr* is True. ≫

```
In[63]:= expr = Exists[{a, b}, Element[a, Reals] && Element[b, Reals],
            ForAll[x, Element[x, Reals], x^3 + 2 x^2 + x + 2
                                         ----------------  == a * x + b]]
                                             x^2 + 1
```

Out[63]= $\exists_{\{a,b\},a\in\mathbb{R}\wedge b\in\mathbb{R}} \forall_{x,x\in\mathbb{R}} \dfrac{x^3+2x^2+x+2}{x^2+1} = a\,x+b$

```
In[64]:= Reduce[expr]
```

Out[64]= True

```
In[65]:= SolveAlways[x^3 + 2 x^2 + x + 2
                     ----------------  == a * x + b, x]
                         x^2 + 1
```

Out[65]= $\{\{a\to 1,\ b\to 2\}\}$

```
In[66]:= Simplify[x^3 + 2 x^2 + x + 2
                  ----------------]
                      x^2 + 1
```

Out[66]= $x+2$

```
In[67]:= Remove[expr]
```

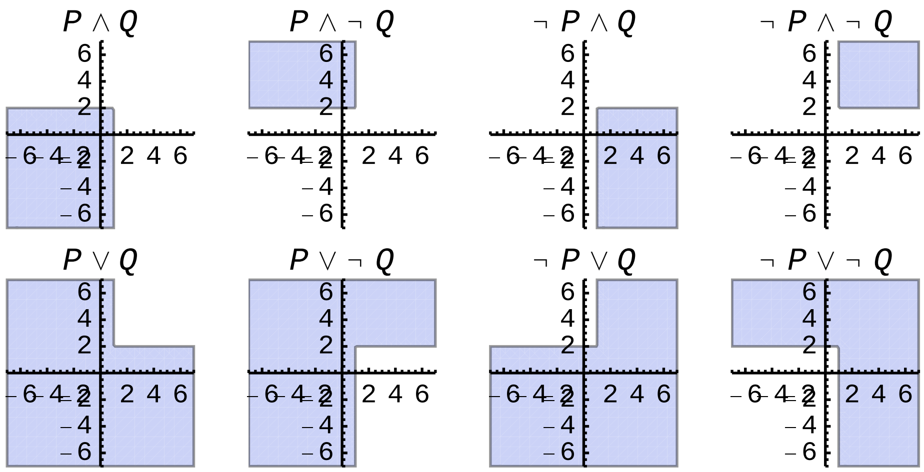
Logique, représentations graphiques

```
In[68]:= propP = x < 1;
         propQ = y < 2;
         cond[donnees_List] :=
           Flatten[
            {And @@@ #, Or @@@ #} &[Tuples[{#, Not[#]} & /@ donnees]]
           ];
         cond[{propP, propQ}]
```

```
Out[71]= {x < 1 ⋀ y < 2, x < 1 ⋀ y ≥ 2, x ≥ 1 ⋀ y < 2, x ≥ 1 ⋀ y ≥ 2,
          x < 1 ⋁ y < 2, x < 1 ⋁ y ≥ 2, x ≥ 1 ⋁ y < 2, x ≥ 1 ⋁ y ≥ 2}
```

```
In[72]:= GraphicsGrid[
          Partition[
           MapThread[
            RegionPlot[
              #1,
              {x, -7, 7},
              {y, -7, 7},
              PlotLabel → #2,
              Axes → True,
              Frame → False,
              PlotRange → {{-7, 7}, {-7, 7}}
             ] &,
            {cond[{propP, propQ}], cond[{P, Q}]}],
           4
          ]
         ]
```

Out[72]=



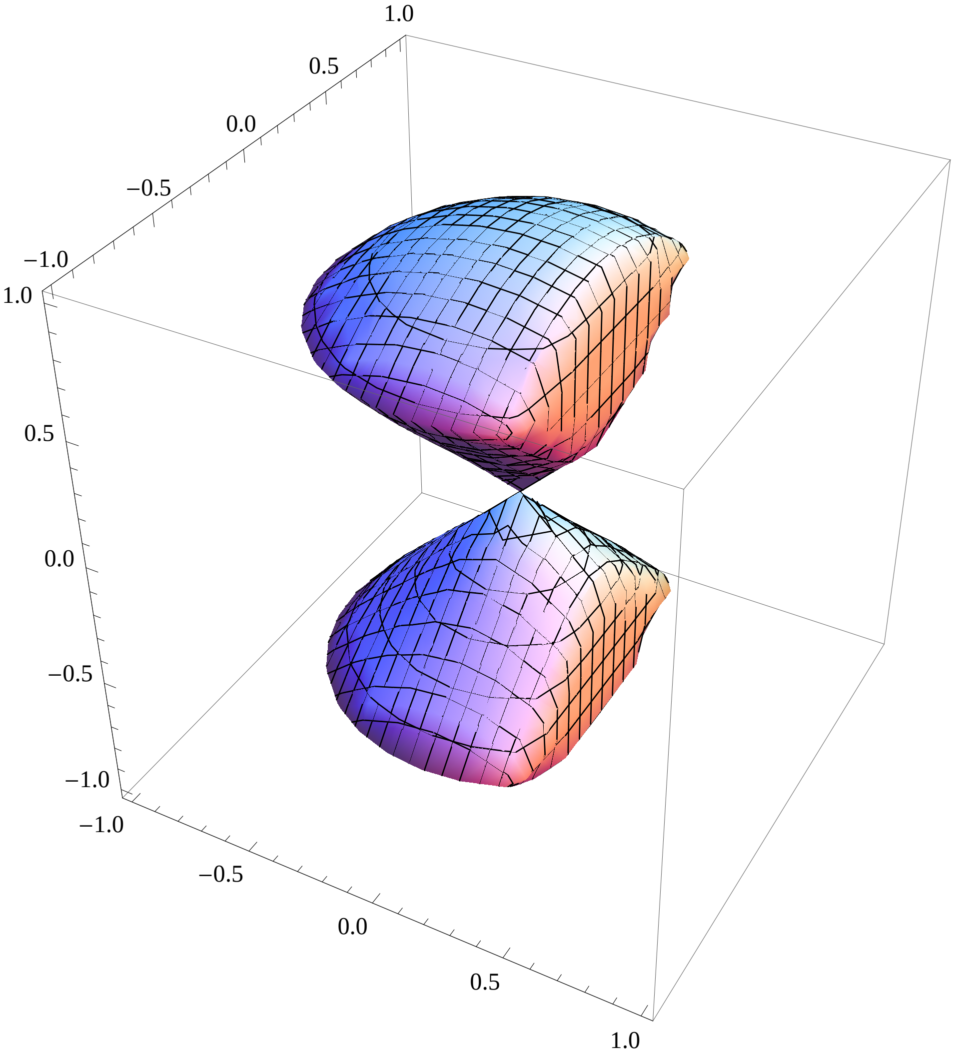
```
In[73]:= Clear[propP, propQ]
```

In[74]:= **RegionPlot3D**[

$x^2 + y^2 + z^2 < 1 \,\&\&\, x^2 + y^2 < z^2 \,\&\&\, x \leq \frac{1}{3}$,

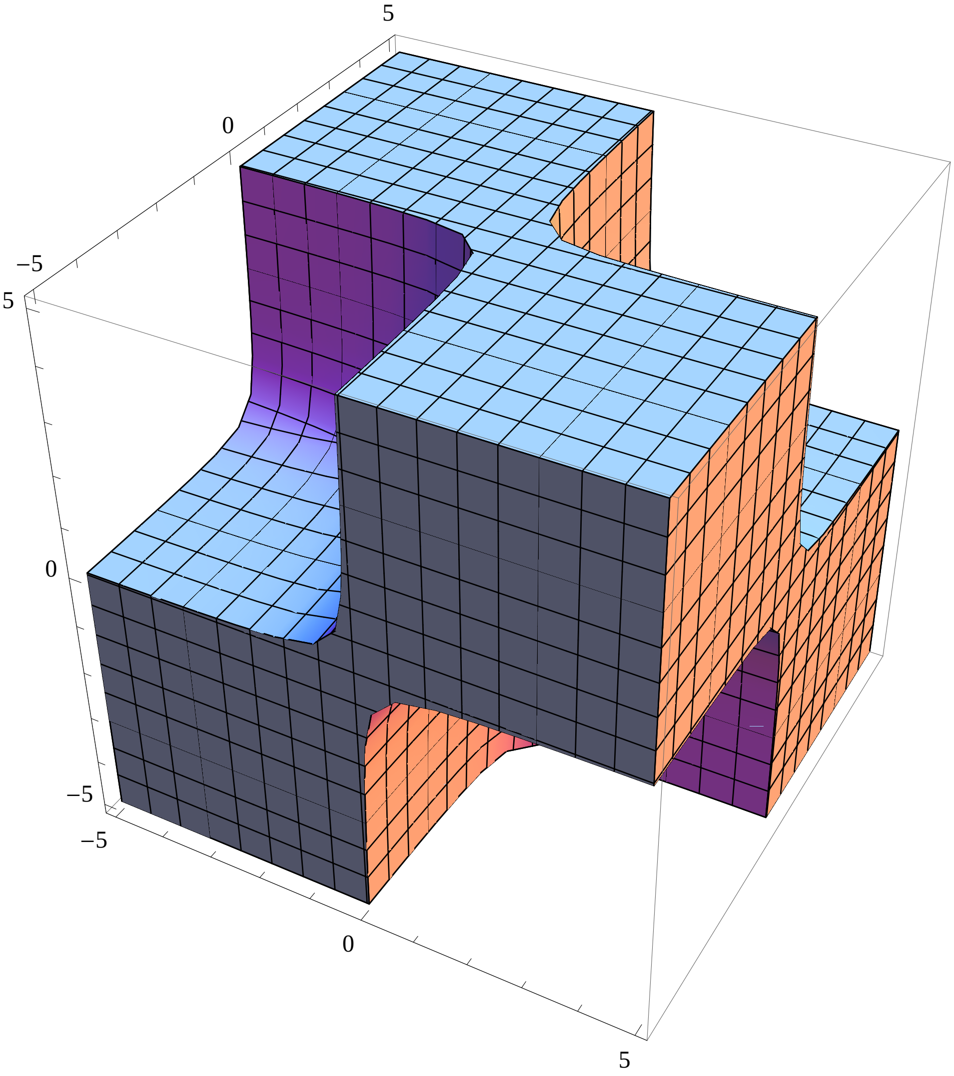
{x, -1, 1},
{y, -1, 1},
{z, -1, 1}
]

Out[74]=



```
In[75]:= RegionPlot3D[
          x * y * z ≤ 1,
          {x, -5, 5},
          {y, -5, 5},
          {z, -5, 5}
         ]
```

Out[75]=



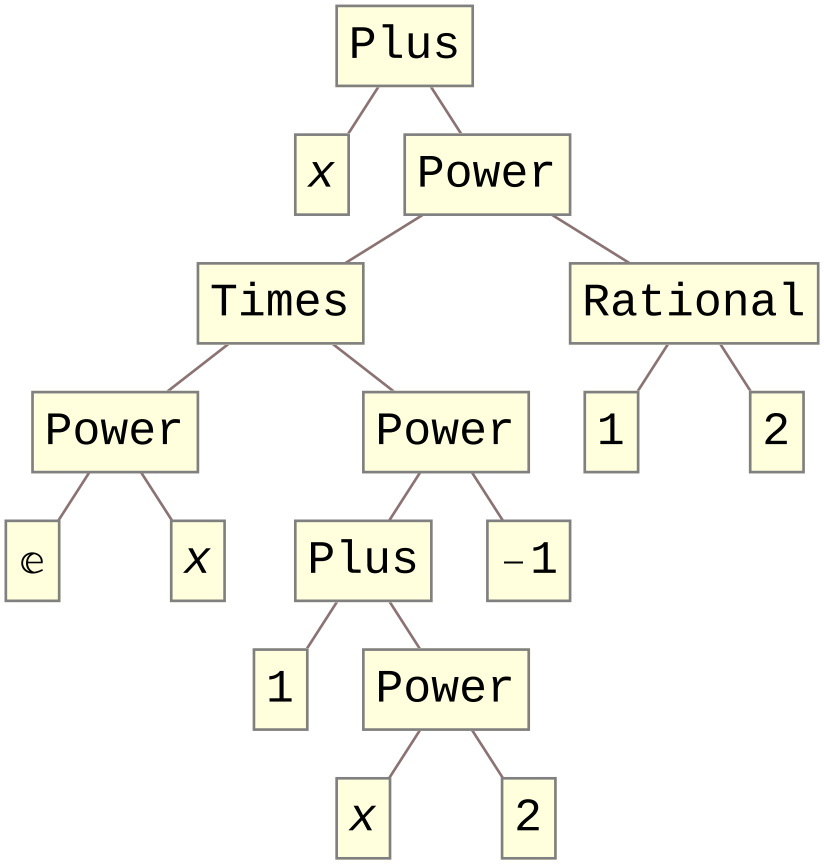
Formes algébriques

Arbre

In[76]:= `f[x_] :=` $x + \sqrt{\frac{E^x}{x^2 + 1}}$

```
In[77]:= TreeForm[f[x]]
```

Out[77]//TreeForm=



Application à la recherche de l'ensemble de définition d'une fonction numérique

```
In[78]:= existinverse[f_] :=
             1 / # != 0 & /@ Extract[
                 f,
                 Position[
                     f,
                     1/_ | _^_Integer?Negative
                 ]
             ];

         existtan[f_] :=
             (Cos @@ # != 0) & /@ Extract[
                 f,
                 Position[
                     f,
                     Tan[_]
                 ]
             ];

         existcotan[f_] :=
             (Sin @@ # != 0) & /@ Extract[
```

```
        f,
        Position[
            f,
            Cot[_]
        ]
    ];

existsec[f_] :=
    (Cos @@ # != 0) & /@ Extract[
        f,
        Position[
            f,
            Sec[_]
        ]
    ];

existcsc[f_] :=
    (Sin @@ # != 0) & /@ Extract[
        f,
        Position[
            f,
            Sec[_]
        ]
    ];

existpuis[f_] :=
    # >= 0 & @@@ Extract[
        f,
        Position[
            f, Sqrt[_] | _^_Rational?Positive
        ]
    ];

existpuisln[f_] :=
    # > 0 & @@@ Extract[
        f,
        Position[
            f,
            Log[_] | _^_Rational?Negative
        ]
    ];

conditionsexistence[f_, x_] :=
    Flatten[
        {    existinverse[#],
            existpuis[#],
            existpuisln[#],
            existtan[#],
            existcotan[#],
            existsec[#],
            existcsc[#]
        } &@f
    ];
```

```
existence[f_, x_] :=
    Reduce[
        Reduce[
            And @@ conditionsexistence[f, x],
            x,
            Reals
            ],
            x,
            Reals
    ] /.
   {C[k_Integer] -> 0, ArcSin[a_] -> arcsin[a], ArcCos[a_] -> arccos[a]};
```

In[87]:=

$$\mathtt{f[x\_] :=} \sqrt{\mathtt{x} + \frac{1}{\sqrt{\mathtt{x} + \mathtt{Log}\left[\frac{\mathtt{E}^{\mathtt{x}}}{\mathtt{E}^{\mathtt{x}}-2}\right]}}}$$

In[88]:=

```
conditionsexistence[f[x], x]
```

Out[88]=

$$\left\{e^x - 2 \neq 0,\ x + \frac{1}{\sqrt{x + \log\left(\frac{e^x}{e^x-2}\right)}} \geq 0,\ \frac{e^x}{e^x - 2} > 0,\ x + \log\left(\frac{e^x}{e^x - 2}\right) > 0\right\}$$

In[89]:=

```
existence[f[x], x]
```

Out[89]= $x > \log(2)$