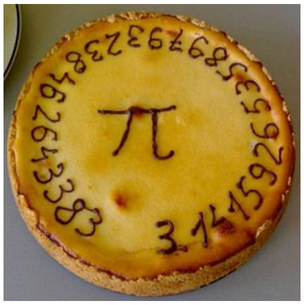


Rational and Irrational Numbers; Number π

Students will be able to:

Content Objective: Define rational and irrational numbers
Language Objective: Discuss how to classify numbers as rational or irrational.

I) Rational numbers

a) Make the following divisions.
b) Which remark could you make?

10 | 2   16 | 3   22 | 7   50 | 7   1000 | 81

Notation: 1.33333333… will be written $1.\overline{3}$

c) Write $\frac{16}{3}, \frac{22}{7}, \frac{50}{7}, \frac{1000}{81}$ with this new notation.

II) Rational and irrational numbers

Rational numbers	Irrational numbers
Definition: Numbers that can be written as a fraction ($\frac{a}{b}$ with a and b as whole numbers). **Property**: They can be written as a decimal (terminating) or with an infinite repeating pattern. Examples: a) $\frac{16}{3}, \frac{22}{7}, \frac{50}{7}, \frac{1000}{81}$ b) 40.5	**Definition**: Numbers that can't be written as a fraction. **Property**: Numbers that never end and never repeat. Examples: a) $\sqrt{2}$ **b)** π c) $7\sqrt{2}$

III) Number Pi (π)

The number π starts as 3.14159265... and continues without end and any periodicity.

The number π has fascinated mathematicians as it didn't obey to criteria of reason. As a matter of fact, π is not a rational, that is to say that it can't be written as the quotient of a whole number by another whole number, like the number $\sqrt{2}$ for instance.

For example, π decimals don't follow a periodic development as any other rational number (like $\frac{2}{3}$ for example: $\frac{2}{3}$ = 0,6666666……).

However, it has an additional strangeness that $\sqrt{2}$ does not possess.

$\sqrt{2}$ is the solution to the algebraic equation $x^2 = 2$.

But the number π is not the solution to any algebraic equation (equation with whole number coefficients), *π is transcendental* (it is beyond reason)!_

That is why it has questioned many philosophers.

Exercises:

1. Which of the following is a rational number?

A. π
B. $\sqrt{2}$
C. 17.58
D. 21
E. $1.\overline{35}$
F. $\frac{237}{45}$

2. Determine whether each fraction is a terminating or repeating decimal.
Write the fraction in the correct box in the answer column.

$\frac{6}{7}$ $\frac{17}{8}$ $\frac{2}{13}$ $\frac{34}{16}$ $\frac{5}{24}$

Terminating	Repeating

3. Which fractions are equivalent to $1.\overline{27}$? Check all that apply.

A. $\frac{9}{7}$

B. $\frac{12}{5}$

C. $\frac{14}{11}$

D. $\frac{28}{19}$

E. $\frac{42}{33}$

F. $\frac{60}{27}$

6. A ☐ D ☐
B ☐ E ☐
C ☐ F ☐

4. True or False. Write T if the statement is correct and F is the statement is incorrect.

_____________ Rational numbers can be written as a fraction, or as a decimal, or with an infinite repeating pattern.

_____________ Irrational numbers cannot be written as a fraction.

_____________ π is an irrational number because its value 3.14159… does not terminate and has no periodicity.

_____________ A rational number is a number that cannot be written as a fraction.

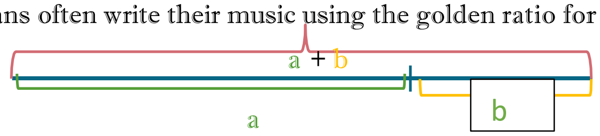
5. Jonathan believes that the difference below results in an irrational number. Is Jonathan correct? Explain.

$$\frac{\sqrt{36}}{3} - \frac{\sqrt{16}}{4}$$

6. Write as a fraction: a) $0.\overline{5}$ b) $2.\overline{37}$

<u>History:</u>

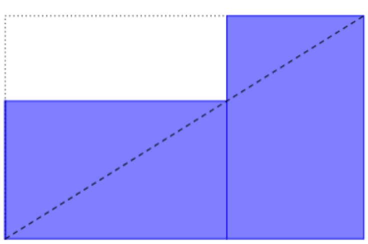
7. The golden ratio can be defined by the Greek symbol "phi" shown below, where a and b represent the lengths of a line segment. It has been said that the golden ratio is considered to make some of the most aesthetically pleasing graphic designs, art, and architecture. Popular bands we know today use the golden ratio to design their logos and musicians often write their music using the golden ratio for note frequency without even realizing it.

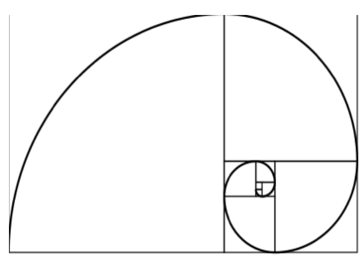


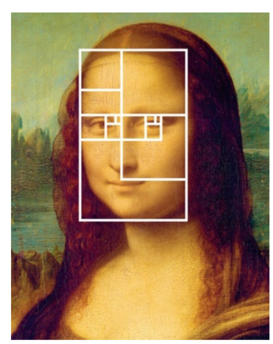
$$\boldsymbol{\varphi} = \frac{\boldsymbol{a+b}}{\boldsymbol{a}} = \frac{\boldsymbol{a}}{\boldsymbol{b}} \qquad \boldsymbol{\varphi} = \frac{\boldsymbol{1+\sqrt{5}}}{\boldsymbol{2}} \qquad \varphi \approx 1{,}61\ldots..$$

Is the value of the golden ratio a rational or irrational number? Explain your reasoning.

Golden ratio and shapes….







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